Appendix E: Temperature Measurement System

The goal is to measure the temperature of some system. The ability to do so accurately and with the required resolution depends on a variety of factors. The calibration report from Lake Shore (or any calibration facility) is only the first step in determining the accuracy of the temperature measurement in the end-user's system.

A more quantifiable term than accuracy is total uncertainty of the measurement. This is simply the measurement itself and an estimate of all the errors of the measurement. Smaller errors are considered more accurate. The first step in estimating the errors in a customer system is the calibration itself. Essentially, a calibration is a series of resistance or voltage measurements of an unknown sensor and a corresponding measurement of an established temperature. By accounting for all the uncertainties of the measurement (installation, instrumentation, etc.) a total uncertainty is estimated. The actual accuracy a customer can expect will depend on this and other factors:

1. Design errors: *Can the system be measured by the sensor?*

These are errors of design and happen prior to sensor installation. For example, whether or not the sensor can be mounted on or near the sample to be measured could be a design error. If it is too far away, there can be thermal lags and offsets due to thermal conductance of the sample. Another example would be using too large a sensor to measure small samples. The thermal mass of the sensor could bias the temperature of the sample.

Design errors also apply to the physical construction of the sensor. This affects the reproducibility of the sensor over thermal cycling. Some sensors are more fragile than others and more prone to physical damage (for example carbon-glass RTDs).

2. Installation and environment errors: *Does the interaction of the sensor and system disturb the measurement?*

This would include installation errors and environmental effects. If leads are not properly heat sunk, they will introduce a heat load into the sensor. This affects the sensor's measurement and can also affect the sample. It can bias the reading of temperature as well as directly affect the temperature if the heat leak is great enough. Other interactions include thermal radiation, magnetic fields, and radiation.

3. Operation and instrumentation errors: *Does the instrumentation introduce errors to the measurement?*

Instrumentation is a crucial component to the total quality of the measurement. The choice of 2-lead or 4-lead measurements, excitation currents, instrument resolution, and accuracy all affect the measurement. Additionally, grounding errors, RF noise coupling, and thermal EMFs can introduce noise to the measurement.

Error terms can be classified into two classes:

Type A, (or random): Errors that can be evaluated by statistical methods.

Type B, (or systematic): Errors that can be evaluated by other means.

Most random errors are the result of instrumentation: uncertainty in the current source and voltage measurements. Other random errors are the actual assignment of a temperature (transferring ITS-90 or PLTS-2000), and interpolation errors. Design, installation, and environmental errors are systematic. For example, sensors in magnetic fields will create an offset to the measurement. This offset can be estimated from prior information or directly measured by other means (isothermal measurements with and without field). RF noise can also cause both random errors (adds to current noise) and systematic errors since at ultra-low temperatures the added noise can self-heat the sensors causing a systematic offset.

Installation

2-lead vs. 4-lead installations can lead to significant measurement errors. Even with a properly installed temperature sensor, poor thermal design of the overall apparatus can produce measurement errors. Installation issues are addressed in Appendix C: Sensor Packaging and Installation, along with detailed installation instructions for specific Lake Shore sensors.

Environmental concerns

Temperature sensors can be affected by changes in the environment. Examples include magnetic fields, ionizing radiation, or changes in the pressure, humidity, or chemistry of the environment. The most common are magnetic field and radiation-induced errors. These effects have been discussed previously. These environmental effects will create a systematic bias in the temperature measurement.

Instrumentation

2-lead versus 4-lead

The measurement of resistance and diode temperature sensors requires passing a current through the temperature sensor to produce a sensor voltage that can be measured. The simplest resistance or voltage measurement configuration is a current source connected to the temperature sensor with a voltmeter connected to the current leads as shown in Figure 1. The current source can be represented as an ideal current source (I_s) in parallel with a shunt resistance, R_s. The voltmeter, normally a digital multimeter (DMM) can be modeled as an ideal voltmeter (V_{in}) in parallel with an input impedance, R_{in}.

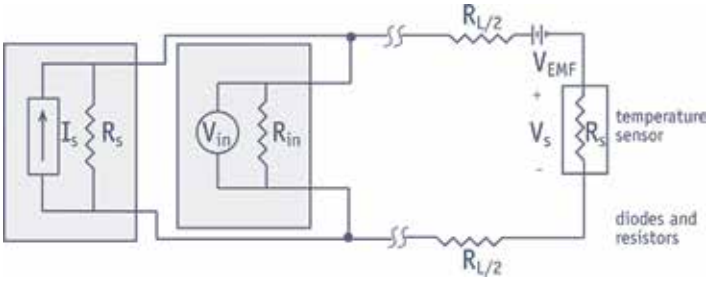


Figure 1—2-lead resistance measurement

The dominant source of error in a 2-lead resistance measurement is usually the resistance of the lead wires connecting the current source to the temperature sensor. In a cryogenic environment, the flow of heat down the leads of the cryostat is of critical concern due to the potential for sensor element heating. Normally, wire of small diameter and significant resistance per unit length is preferred to minimize this heat flow. Consequently, the resulting lead resistance can become a significant percentage of the resistance measured. The wire also has its own temperature sensitivity of resistance.

The equivalent error the lead resistance represents depends on the sensor type and sensor sensitivity. The 100 Ω platinum RTD has a nominal resistance of 100 Ω at 273.15 K (0 °C). The IEC 751 standard for the temperature sensitivity for platinum RTDs is 0.385 Ω/K between 273.15 K and 373.15 K (0 °C to 100 °C). Both the magnitude of the resistance and the temperature sensitivity are relatively small numbers, especially when the lead resistance may be several ohms. A 10 Ω lead resistance would result in a positive 26 K error in this temperature range (10 Ω/0.385 Ω/K = 26 K). The effect of lead resistance becomes even greater as the temperature decreases, since the temperature sensitivity (dR/dT) of platinum sensors decreases with decreasing temperature.

Additionally, it is not uncommon for the internal lead resistance of the current leads (parasitic resistance) of a germanium or carbon-glass sensor to be as much as 10% to 20% or more of the sensor 4-lead resistance. Consequently, the 4-lead calibrated resistance-temperature data is of little use for a 2-lead measurement and the temperature error associated with 2-lead resistance measurements for germanium and carbon-glass is almost always extremely large. The parasitic resistance for Cernox® temperature sensors, due to having common current and voltage contact, is extremely small. Even still, the low temperature error due to lead resistance can be at least 3 mK for 100 Ω of lead resistance. Since lead wire has its own temperature dependence, the error could be much larger. Table 1 shows typical error with 2-lead measurement.

In order to eliminate the effects of lead resistance, a 4-lead measurement (Figure 2) is normally used. Two of the leads, I+ and I–, supply current to the sensor, while the other two leads, V+ and V–, are used to eliminate the effect of lead resistance by measuring the voltage at the sensor voltage leads (4-lead sensor) or directly at the device leads (2-lead sensor). The reason this measurement scheme works is that the IR drop in the current leads is not measured, and the voltage drop in the voltage leads is extremely small due to the very small current required by the voltmeter (picoamperes or less) to make the voltage measurement.

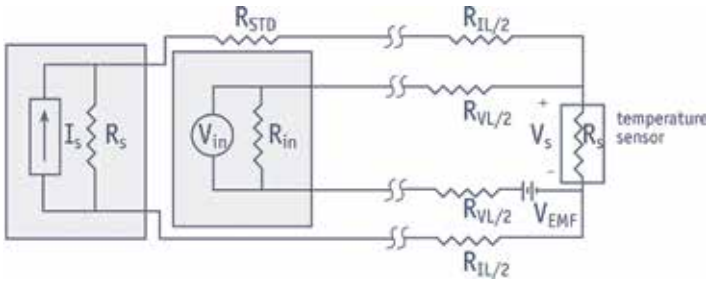


Figure 2—4-lead resistance measurement

A diode temperature sensor measurement requires a fixed 10 µA current source and a voltmeter. As with resistance measurements, the dominant source of error in a 2-lead diode measurement is often the lead resistance. A 100 Ω lead resistance will result in a 1 mV voltage error at a current of 10 µA. The Lake Shore DT-400 Series silicon diode temperature sensors have an average sensitivity of approximately –26 mV/K below 30 K, resulting in a temperature error of 40 mK (1 mV/26 mV/K = 0.038 K); above 30 K the sensitivity is approximately –2.3 mV/K, resulting in error exceeding 400 mK (1 mV/2.3 mV/K = 0.435 K). Consequently, unless the lead resistance can be reduced in magnitude or the resultant error can be tolerated, a 4-lead measurement is recommended.

Table 1—Typical errors for Cernox® 1070 resistor with lead resistance at 100 Ω (50 Ω each lead)

	R (Ω)	dR/dT (Ω/K)	ΔT (mK)
4.2 K	52444	-33321	-3.0
20 K	2851.6	-200.70	-498
100 K	317.44	-4.3908	large
300 K	68.949	-0.3052	very large

Voltmeter input impedance

The voltmeter input impedance is generally not a problem in 2- or 4-lead measurements. It is not uncommon for today's voltmeters to have a 109 Ω or 1010 Ω input impedance in the voltage ranges of interest, which is large when compared to the temperature sensor resistance. Consequently, virtually no current will be shunted from the temperature sensor into the voltage measurement circuitry at these input impedance levels. A voltmeter input impedance of 109 Ω would produce only a 0.0001% error in a 1000 Ω resistance measurement.

Current source output impedance

The output impedance of a good current source is also not ordinarily a problem in either 2- or 4-lead measurements, for the same reason. If the output impedance is not large compared to the sensor resistance, then a known series resistor should be placed in one of the current paths, and the current to the sensor should be measured by measuring the voltage across the known standard resistance.

Thermoelectric and zero offset voltages

Voltages develop in electrical conductors with temperature gradients when no current is allowed to flow (Seebeck effect). Thermoelectric voltages appear when dissimilar metals are joined and joints are held at different temperatures. Typical thermoelectric voltages in cryogenic measurement systems are on the order of microvolts.

This effect can be minimized by a few steps. The same material should be used for conductors whenever practical, and the number of connections, or joints, in the measurement circuit should be minimized. Low thermal EMF solder can also be used (cadmium-tin solder has a lower thermal EMF than tin-lead solder by a factor of ten).

In addition to thermal offset, the instrumentation can have a zero offset (the signal value measured with no input to the measuring instrument). The zero offset can drift with time or temperature and is usually included in the instrument specifications.

The total offset voltage can be measured by reversing the current. When reading the voltage with the current in the forward direction, the voltmeter will read:

$$V^{+} = V_S + V_{EMF} \qquad \text{Eqn. 1}$$

where V_S is the actual voltage reading of the sensor, and V_{EMF} is the lead thermal EMFs. When the current is reversed, the voltmeter will read

$$V^{-} = -V_S + V_{EMF} \qquad \text{Eqn. 2}$$

When the current is reversed, the voltage due to the sensor reverses sign while the thermal EMFs do not. The true voltage (V) across the sensor is

$$V = (V^{+} - V^{-})/2 = V_S \qquad \text{Eqn. 3}$$

By averaging the forward and reverse current voltage measurements, the error in the voltage measurement due to thermal EMFs is eliminated.

Diode measurements do not allow current reversal. The value of the offset voltage can be estimated by shorting the leads at the diode and measuring the offset voltage with zero excitation current at operating temperature.

Thermal EMFs in the sensor leads and connections do not have as big an effect on diode measurements as they do on resistance measurements, since the diode signal levels are much larger (typically a few tenths of a volt at room temperature to several volts at 4.2 K).

Grounding

Signal grounding is important to the stability and repeatability of measurements. A measurement system that includes sensors, instruments, cabling, and possibly computer interfacing requires careful grounding.

Improper grounding of instruments or grounding at multiple points can allow current flows which result in small voltage offsets. The current flow through ground loops is not necessarily constant, resulting in a fluctuating voltage. Current can flow in the ground loop as it acts as a large aperture for inductive pickup. Also, current can result if there is a potential difference due to multiple grounds.

As each instrument handles grounding differently, it is important to carefully read your instrument manual for grounding suggestions. The grounding and isolation is handled differently in the Model 370 than in other Lake Shore instruments, since it is used for ultra-low temperature measurements. Ideally, there should be one defined ground for the measurement, and the cryostat is the best choice. Realistically, however, there are many instruments, wiring, and pumps attached to the cryostat. Each instrument may have its own ground. Simply attaching ground straps may create more ground loops.

Books on grounding and shielding can help to identify and eliminate both ground loops and electromagnetic noise.

Reducing AC signal interference (RF noise)

Signal leads and cables are very susceptible to interference from unwanted AC signals in the RF frequency range. They act like antennas and pick up noise from computers, monitors, instrumentation, radio broadcasts, and other sources. Signals are either inductively coupled or capacitively coupled. The induced signals circulate as noise current in the measurement leads and distort measurements. There are other concerns when diodes are used as the sensing element, as discussed in the next section.

There are several ways to reduce the effect of AC signals. First, when possible, remove or shield the source of unwanted signals. Second, make each pair of signal leads as bad an antenna as possible. This can be accomplished by keeping them short and using twisted leads. Twisting reduces loop area to make leads that are prone to picking up noise smaller targets to electromagnetic signals. Twisting also helps to cancel unwanted signals in leads that are prone to transmit noise. In a typical 4-lead measurement, the current leads should be twisted together and the voltage leads should be twisted together. Third, put a conductive shield around all the leads to divert electric field signals and prevent capacitive coupling into the leads. Tie the shield to the ground closest in potential to the measurement. Many Lake Shore instruments provide a shield pin on the sensor connector for this purpose. The shield should be tied only at the instrument. Attaching at any other point can cause ground loops that were previously discussed.

In cases where shielding is not enough, filtering the unwanted signals can be considered. It is very difficult to add a filter to a measurement system without changing the measurement. One type of filter that has proven to work is a ferrite bead (see the Accessories section). The bead will act like a high impedance to unwanted high frequency signals and not affect the slow moving desired signals being measured. The Lake Shore 2071 ferrite bead can be clamped around existing wiring.

The greatest concern relates to leads external to the cryostat. Ideally, the cryostat itself acts as the shield for all wiring internal to it. However, it is still possible for cross-talk between different signal leads. In this application Lake Shore recommends Quad-Twist™ cryogenic wire, which has two twisted pairs of phosphor bronze wire that minimize noise pickup and allow proper heat sinking. In extreme cases coaxial cable may be needed, although it is much more difficult to heat sink.

Measurement errors in diode thermometers due to AC interference

Wiring techniques are especially important when using diode thermometers in a measurement system. Noise currents produce a shift in measurement. Because diodes have a nonlinear voltage response to the changing current, the shift is seen as a lower measured voltage corresponding to a higher measured temperature. The temperature error in noisy systems can be as high as several tenths of a kelvin.

The following equation can be used to estimate the temperature shift with DT-470 silicon diodes over the range $0 < V_{RMS} < 40$ mV and $30 < T < 300$ K. The temperature errors tend to decrease at temperatures below 30 K (ΔT in K, T in K, and V_{RMS} in mV).

$$\Delta T = 2.7768 \cdot T^{-1.11953} \cdot V_{RMS}^{2.01803} \qquad \text{Eqn. 4}$$

There are two simple techniques that can be used to determine if this problem is present in the measuring system. The first is to connect a 10 µF capacitor in parallel with the diode to act as a shunt for any induced AC currents. The capacitor must have low leakage current so it does not alter the DC current through the diode. If the DC voltage reading across the diode increases with the addition of the capacitor, AC noise currents are present. The second method involves the measurement of the AC voltage across the diode. While an oscilloscope is the logical choice for looking at AC signals, many do not have the sensitivity required and often introduce unwanted grounds into the system and compound the problem. An AC voltmeter should be used.

Lake Shore instrumentation includes a 1 µF capacitor across the current source in order to minimize the effects of noise related to power line frequency. A 0.1 µF capacitor in parallel with a 30 pF to 50 pF capacitor at the voltage measurement input are used to minimize the effects of AC-coupled digital noise. The obvious disadvantage of the addition of AC filtering is that it slows down the response time of the measurement system.

Effect of current source accuracy

Diode temperature sensors—Measurement accuracy of diode sensors is not as strongly dependent upon the current source accuracy as is the case with resistance temperature sensors. Diode sensors possess a nonlinear forward current-voltage characteristic. Consequently, the forward voltage variation with changing current for diodes is smaller than for resistance temperature sensors, which have linear current-voltage characteristics.

Below 30 K, the sensitivity (dV/dT) of Lake Shore diode temperature sensors increases by an order of magnitude over sensitivities at higher temperatures. The slope (dV/dI) of the I-V curves (Table 2) stays relatively constant. Both characteristics further reduce the effect of any change in forward bias current on temperature measurement accuracy.

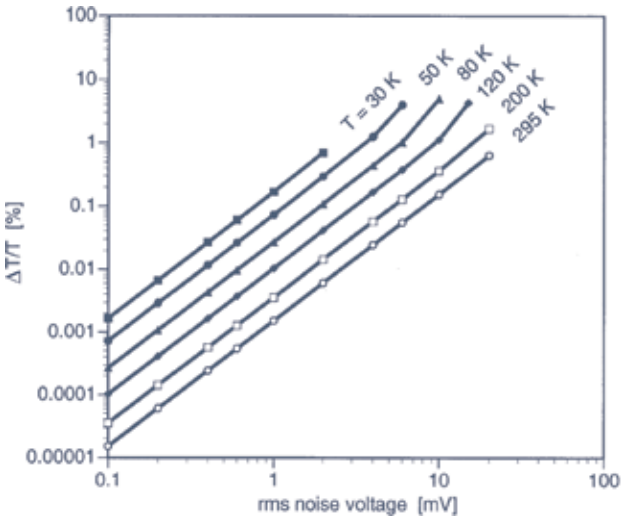


Figure 3—Calculated temperature reading shifts due to voltage noise across a Lake Shore Model DT-470 silicon diode temperature sensor

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Table 2 – Approximate dV/dI values for the DT-470 sensor

	Approximate dV/dI (Ω)
300 K	3000
100 K	1000
40 K	700
4.2 K	2800

Table 3—Equivalent temperature offsets for the DT-470 diode temperature sensors at selected current source uncertainties

	dV/dI (Ω)	dV/dT (mV/K)	Temperature offset (mK)	
			dI(%)=0.05	dI(%)=0.1
300 K	3000	-2.40	6.5	13
100 K	1000	-2.04	2.5	5
40 K	700	-1.74	2.0	4
4.2 K	2800	-33.6	0.4	1

Lake Shore diode current sources are typically set to 10 µA ±0.1% or better and have a low-pass filter to minimize the effect of AC pickup in the current leads. Resultant errors due to current source inaccuracy are on the order of 10 mK or less for diode sensors.

If the output from a current source is not precisely 10 µA, the resultant error in temperature can be calculated using this relationship between the dV/dT and dV/dI values:

$$\Delta T = ((dV/dI)/(dV/dT))\Delta I \qquad \text{Eqn. 5}$$

Note: dV/dI and dV/dT values are derived at the same temperature T.

In the above expression, $R_d = dV/dI$ and $R_s = V/I$ are the dynamic and static resistances of the temperature sensor. Note that the dynamic and static resistances of an ohmic sensor are equal. Results shown in Table 3.

Resistance temperature sensors—for resistance sensors, an error in current measurement is inversely related to the resultant measurement error of resistance:

$$R - \Delta R = V/(I + \Delta I) \qquad \text{Eqn. 6}$$
$$\approx (V/I)(1 - \Delta I/I)$$
$$= R - R(\Delta I/I)$$

where I is the current setting, ΔI is the variation from that setting, and $\Delta R = R\Delta I/I$.

The temperature error, ΔT, due to current source uncertainty, ΔI(%), is

$$\Delta T = \Delta R/(dR/dT) \qquad \text{Eqn. 7}$$
$$= R(\Delta I/I)/(dR/dT)$$
$$\Delta T = \Delta I(\%)/ [(100/R)(dR/dT)]$$

where $\Delta I(\%) = 100\ \Delta I/I$

All Lake Shore resistance current sources are typically set to 0.01%. For example (Table 4), temperature errors for a platinum resistance sensor near room temperature due to the current source can approach 36 mK and diminish to less than 10 mK below 100 K.

Table 4—Equivalent temperature offsets for selected resistance sensors at selected voltmeter and current source uncertainties

	T (K)	R (Ω)	dR/dT (Ω)/K	Temperature offset (mK)	
				dV(%)=0.01 dI(%)=0.01	dV(%)=0.05 dI(%)=0.05
PT-100	300	110.452	0.388	28.5	142.5
	100	29.987	0.411	7.3	36.5
	40	5.938	0.291	2.0	10
CGR-1-1000	300	6.21021	-0.0047	132	660
	100	9.66389	-0.0465	20.8	104
	40	16.8227	-0.3211	5.2	26
	4.2	964.19	-842.21	0.1	1
CX-1050	300	50.1	-0.165	30.4	152
	100	154.62	-1.55	10.0	50
	40	376.1	-8.9	4.2	21
	4.2	4596	-1867	0.2	1
GR-200A-1000	100	4.95987	-0.0469	10.6	53
	40	18.7191	-0.844	2.2	11
	4.2	981.026	-451.3	0.2	1

Effect of voltage measurement accuracy

Diode temperature sensors—The effect of voltage measurement accuracy on resultant temperature measurement is not difficult to calculate, provided that diode sensitivity is known for the temperature of interest. The potential temperature error, ΔT_V is

$$\Delta T_V = \Delta V/[dV/dT] \qquad \text{Eqn. 8}$$

Table 5 illustrates potential temperature error due to the voltage measurement.

Table 5—Equivalent temperature offsets for the DT-470 diode temperature sensor at selected voltmeter uncertainties

	T (K)	V (V)	dV/dT (mV/K)	Temperature offset (mK)	
				ΔV(%)=0.01	ΔV(%)=0.05
DT-470	300	0.51892	-2.40	21.6	108
	100	0.97550	-2.04	47.8	239
	40	1.08781	-1.74	62.5	313
	4.2	1.62602	-33.6	4.8	24

Resistance temperature sensors—for positive temperature coefficient resistors such as platinum or rhodium-iron, the potential temperature error, ΔT_R, is

$$\Delta T_R = \Delta R / [dR/dT] \qquad \text{Eqn. 9}$$

$$= [\Delta V/I] / [dR/dT]$$

since from Ohm's law, $\Delta V = I\Delta R$.

But ΔV (%) = $100\Delta V/V$; therefore

$$\Delta T_R = [V\Delta V\ (\%)\ /100I] / [dR/dT] \qquad \text{Eqn. 10}$$

$$= [\Delta V\ (\%) \cdot R/100] / [dR/dT]$$

$$\Delta T_R = \Delta V\ (\%) / [(100/R)\ (dR/dT)], \text{ and}$$

$$\Delta T_R = \Delta I(\%)/ [(100/R)(dR/dT)]$$

The temperature offsets in Table 4 are calculated using both of the above equations.

This is not surprising, as we are dealing with Ohm's Law and a linear system.

Self-heating

Any difference between the temperature of the sensor and the environment the sensor is intended to measure produces a temperature measurement error or uncertainty. Dissipation of power in the temperature sensor will cause its temperature to rise above that of the surrounding environment. Power dissipation in the sensor is also necessary to make a temperature measurement. Minimization of the temperature measurement uncertainty thus requires balancing the uncertainties due to self-heating and output signal measurement.

Self-heating is really a combination of sensor design and instrumentation. The primary reason for self-heating offsets at low temperatures is the thermal boundary resistance between the active sensor element and its surroundings. The thermal boundary resistance has a very strong inverse cube relationship with temperature. This forces the instrumentation to be capable of sourcing a small excitation and measuring a small (voltage) signal. The optimum excitation power will be a function of sensor, resistance, and temperature.

Lake Shore temperature controllers each have different excitation currents for NTC RTDs which effectively defines the minimum temperature range of the instrument-sensor combination.

An estimate of the self-heating error including thermal resistance for select sensors and optimum excitation power is found in Table 6 (page 195).

Thermal (Johnson) noise

Thermal energy produces random motions of the charged particles within a body, giving rise to electrical noise. The minimum root mean square (RMS) noise power available is given by $P_n = 4kT\,\Delta f_n$, where k is the Boltzmann constant and Δf_n is the noise bandwidth. Peak-to-peak noise is approximately five times greater than the RMS noise. Metallic resistors approach this fundamental minimum, but other materials produce somewhat greater thermal noise. The noise power is related to current or voltage noise by the relations: $I = [P_n/R_d]^{0.5}$ and $V = [P_n R_d]^{0.5}$. The noise bandwidth is not necessarily the same as the signal bandwidth, but is approximately equal to the smallest of the following:

- π/2 times the upper 3 dB frequency limit of the analog DC measuring circuitry, given as approximately $1/(4\,R_{eff}\,C_{in})$ where R_{eff} is the effective resistance across the measuring instrument (including the instrument's input impedance in parallel with the sensor resistance and wiring) and C_{in} is the total capacitance shunting the input
- $0.55/t_r$ where t_r is the instrument's 10% to 90% rise time
- 1 Hz if an analog panel meter is used for readout
- One half the conversion rate (readings per second) of an integrating digital voltmeter

Calibration uncertainty

Commercially calibrated sensors should have calibrations traceable to international standards. About the best accuracy attainable is represented by the ability of national standards laboratories. Many laboratories provide calibrations for a fee. The calibration uncertainty typically increases by a factor of 3 to 10 between successive devices used to transfer a calibration.

Calibration fit interpolation uncertainty

Once a calibration is performed, an interpolation function is required for temperatures that lie between calibration points. The interpolation method must be chosen with care, since some fitting functions can be much worse than others. Common interpolation methods include linear interpolation, cubic splines, and Chebychev polynomials. Formulas based on the physics of the sensor material may give the best fits when few fit parameters are used.

Use of an interpolation function adds to the measurement uncertainty. The additional uncertainty due to an interpolation function can be gauged by the ability of the interpolation function to reproduce the calibration points. Each calibration can be broken up into several ranges to decrease the fitting uncertainties. Typical uncertainties introduced by the interpolation function are on the order of one tenth the calibration uncertainty.

Combining measurement uncertainties

Estimating the quality of a measurement involves the following steps: 1) identify the relevant sources of measurement uncertainty, 2) change the units of all uncertainties to temperature, and 3) combine all of the uncertainties using the root sum of squares method described later. Examples of source of measurement uncertainties affecting the accuracy, but not the precision of a measurement include offset voltages and calibration uncertainties.

The expected uncertainty of a measurement is expressed in statistical terms. As stated in the Guide to the Expression of Uncertainty in Measurement:

"The exact values of the contributions to the error of the measurement arising from the dispersion of the observations, the unavoidable imperfect nature of the corrections, and incomplete knowledge are unknown and unknowable, whereas the uncertainties associated with these random and systematic effects can be evaluated. ...the uncertainty of a result of a measurement is not necessarily an indication of the likelihood that the measurement result is near the value of the measurand; it is simply an estimate of the likelihood of nearness to the best value that is consistent with presently available knowledge."

The uncertainty is given the symbol u and has the same units as the quantity measured. The combined uncertainty u_c arising from several independent uncertainty sources can be estimated by assuming a statistical distribution of uncertainties, in which case the uncertainties are summed in quadrature according to

$$u_c = \sqrt{u_i^2 + u_2^2 + \ldots + u_i^2 + \ldots + u_n^2} \qquad \text{Eqn. 11}$$

Both random and systematic uncertainties are treated in the same way. Note that both sides of Equation 11 can be divided by the measurement quantity to express the measurement uncertainty in relative terms. Finding statistical data suitable for addition by quadrature can be a problem; instrument and sensor specifications sometimes give maximum or typical values for uncertainties. Two approaches may be taken when dealing with maximum uncertainty specifications. The conservative approach is to use the specification limit value in the combined uncertainty calculation. The less conservative approach is to assume a statistical distribution within the specification limits and assume the limit is roughly three standard deviations, in which case one third of the specification limit is used in uncertainty calculations. The manufacturer may be able to supply additional information to help improve uncertainty estimates. Practical recommendations and procedures for problems related to the estimation of measurement uncertainties are discussed in greater detail by Rabinovich.

Table 6 gives examples of uncertainty calculations for two types of temperature sensors, the DT-470-SD silicon diode sensor, and the CX-1050-AA Cernox® sensor.

When Lake Shore accounts for uncertainties in calibration measurements, all the above issues are taken into consideration, and their contributions are estimated.

References:

ISO/TAG 4/WG 3. *Guide to the Expression of Uncertainty in Measurement, First Edition.* Geneva, Switzerland: International Organization for Standardization, 1992.

S. Rabinovich, *Measurement Errors*, College Park, Maryland: American Institute of Physics, 1993.

Table 6—Combined temperature measurement calculation examples

	DT-470-SD-11		CX-1050-AA	
Temperature, T	80 K		4.2 K	
Mounting environment (N-greased to block)	vacuum		liquid helium	
Static Electrical Resistance, R_s	101,525 Ω (static R_s = V/I)		4920 Ω (static R_s = V/I)	
Dynamic Electrical Resistance, R_d	1000 Ω (dynamic R_d = dV/dI)		4920 Ω (dynamic R_d = dV/dI)	
Excitation current, I	10 µA		1 µA	
Output voltage, V	1.01525 V		4.92 mV	
Dimensionless temperature sensitivity, S_D	-0.1521		-1.71	
Uncertainties due to:	Value used	Temperature uncertainty u_T /T (PPM)	Value used	Temperature uncertainty u_T /T (PPM)
Measurement instrumentation				
(Keithley Instruments 2000 DVM)				
Meter range full scale (FS)	10.00000 V		100.0000 mV	
Voltage accuracy specification (ppm)	±(30+5 FS/V)	521	±(50+35 FS/V)	445
Sensor self-heating				
Thermal resistance	R_t = 1000 K/W	127	R_t = 3500 K/W	4.1
Excitation uncertainty (Lake Shore Model 120-CS)				
Current accuracy specification	u_I/I = 0.05%	32	u_I/I = 0.1%	585
Thermal noise		0.02		0.2
Thermal voltages and zero drift	10 µV	65	0[1]	0
Electromagnetic noise[2]	2 mV	1040	0[1]	0
Calibration uncertainty	0.250 K[3]	3130	4 mK	952
Interpolation uncertainty[4]	313		95.2	
Combined uncertainties (ppm)	3357		1206	

[1] *Eliminated by current reversal*

[2] *Assuming an AC voltage of 2 mV_{rms} is read across the voltmeter terminals—the voltage is converted to an approximate temperature shift*

[3] *Calibration accuracy*

[4] *Assumed to be one tenth the calibration uncertainty*

Estimating self-heating of temperature sensors

Any difference between the temperature of the sensor and the environment the sensor is intended to measure produces a temperature measurement error or uncertainty. Dissipation of power in the temperature sensor will cause its temperature to rise above that of the surrounding environment. Power dissipation in the sensor is also necessary to make a measurement with most temperature sensors (exceptions include thermocouples and optical pyrometers). Minimization of the temperature measurement uncertainty thus requires balancing the uncertainties due to self-heating and output signal measurement. The possibility that other experimental considerations might impose more stringent limitations on the power that can be dissipated in the temperature sensor should also be considered.

Following are two approaches to dealing with the problem of self-heating:

1. Choose an excitation that allows acceptable instrumentation measurement uncertainty and check to make sure self-heating is negligible at one or two points where it is likely to be most significant.

An easy way to check for self-heating is to increase the power dissipation and check for an indicated temperature rise. Unfortunately, this procedure will not work with non-linear devices such as semiconductor diodes. An indication of the self-heating error can be made by reading the diode temperature in both a liquid bath and in a vacuum at the same temperature, as measured by a second thermometer not dissipating enough power to self-heat significantly.

2. Measure the thermal resistance in the temperature range of interest and calculate the optimum operating point.

Examination leads to the conclusion that an increase in the sensor output voltage will result in a decreasing temperature uncertainty, so long as the voltage uncertainty remains constant. This is possible with an ohmic sensor by increasing the excitation current. Unfortunately, a larger excitation will dissipate more power in the temperature sensor, raising its temperature above the surroundings.

The self-heating depends on the excitation power according to the equation

$$\Delta T_{sh} = P_s R_t = I^2 R_e R_t = V^2 R_t / R_e \qquad \text{Eqn. 12}$$

where ΔT_{sh} is the temperature rise due to self-heating, P_s is the power dissipated in the sensor, I is the excitation current, R_e is the electrical resistance, and R_t is the thermal resistance between the sensor and its environment. The thermal resistance is extremely difficult to calculate for all but the simplest cases and is best determined experimentally using the following procedure:

1. Mount the sensor as it will be used on a temperature controlled block or directly in liquid

2. Record the output voltage as a function of excitation current (I-V curve) until significant self-heating is observed (when R_e =V/I is no longer constant)

3. Replot the data as sensor temperature reading versus power dissipated (T versus P),

4. Fit the data with a linear equation of the form $T = T_o + R_t P_s$ to find the thermal resistance, R_t

Thermal resistance values determined from some commercial resistance temperature sensors in common mounting configurations are shown as a function of temperature in Figure 4. The thermal resistance varies with the environment in and around the sensor package (vacuum, gas, liquid), sensor mounting (solder, grease, clamp pressure, epoxy, etc.) and details of sensor construction. The thermal resistances shown in the figure should be used only as a guide with reference to the source papers and preferably measurement on the actual sensor in the temperature range and environment of use. See www.lakeshore.com for additional notes and papers.

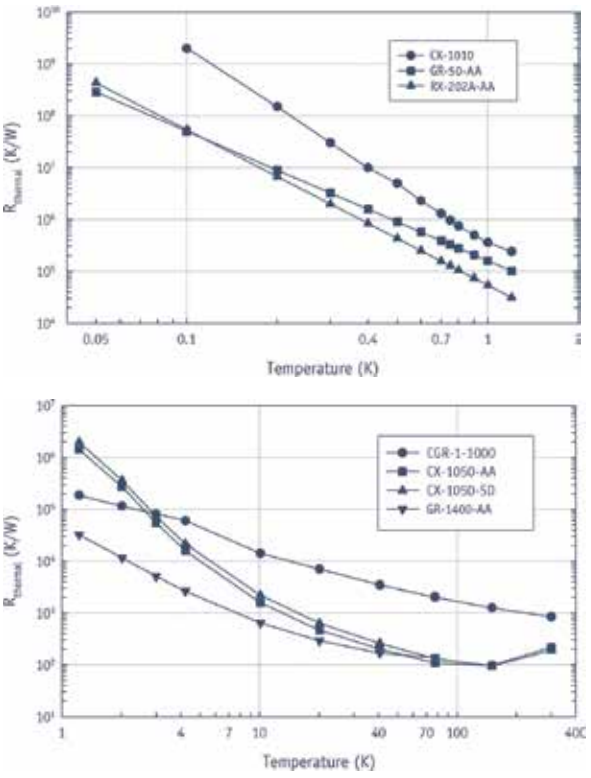


Figure 4—Thermal resistance data for various sensors as a function of T