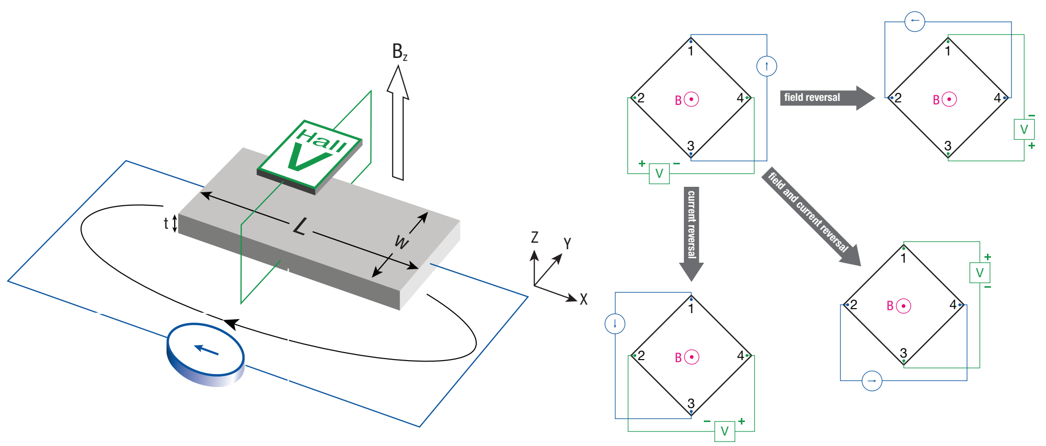
TECHNICAL NOTE

Understanding and minimizing errors that can interfere with Hall Effect Measurements

by Lake Shore Cryotronics

Hall effect measurements are a fundamental tool for semiconductor material characterization. Combined with material resistivity measurements, Hall effect measurements determine important properties of the semiconductor material such as the polarity of the charge carrier, charge carrier concentration, and charge carrier mobility. The challenge with Hall effect voltage measurements is that they can be very small quantities such as microvolts or even lower. This is particularly true if the semiconductor material has a high resistivity and/or low carrier mobility. At these low voltage levels, error terms can be of significant magnitude to have an impact on the measurement of the Hall voltage. For Hall effect measurements, error terms come from a number of sources including errors related to sample construction and sample geometry, inherent errors in metal connections to a sample, voltage errors resulting from non-ideal sample geometries, and thermoelectric errors. This technical note will address the most common of these errors and recommend methods to either eliminate or minimize the errors to achieve quality Hall effect measurements.



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Contact resistance non-linearity and Hall structure error sources

The American Society for Testing and Materials (ASTM) defines test sample geometries for Hall effect measurements in the document, ASTM-F76 Standard Test Methods for Measuring Resistivity and Hall Coefficient and Determining Hall Mobility in Single-Crystal Semiconductors. The two geometries are van der Pauw structures and Hall bar structures. The van der Pauw structures, based on resistivity research conducted by Dr. L. J. van der Pauw are shown in Figure 1.

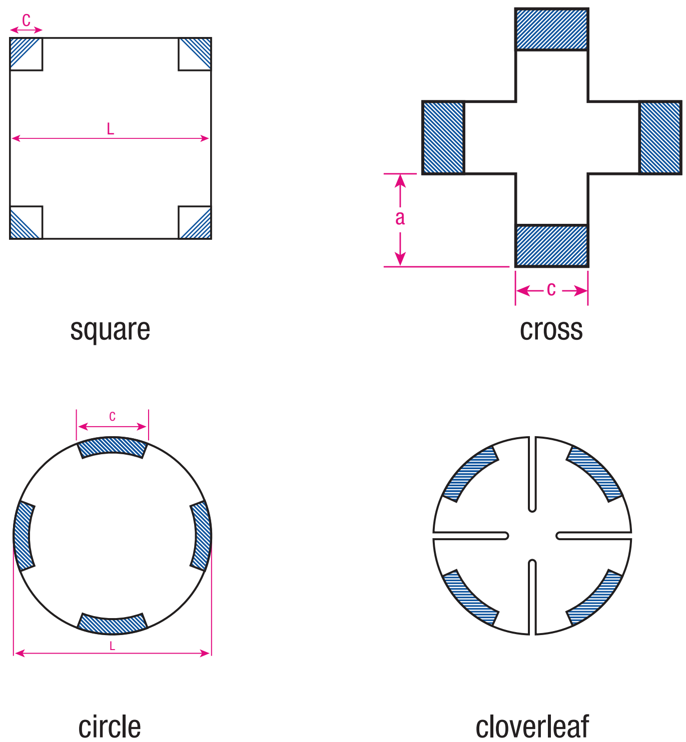


Figure 1 *Van der Pauw sample structures. The electrical contacts are the darkened areas.*

Hall bars offer a more ideal geometry for making Hall effect measurements, since this type of structure enables a flow of constant current density through the long axis of the structure. An example of a Hall bar structure is shown in Figure 2.

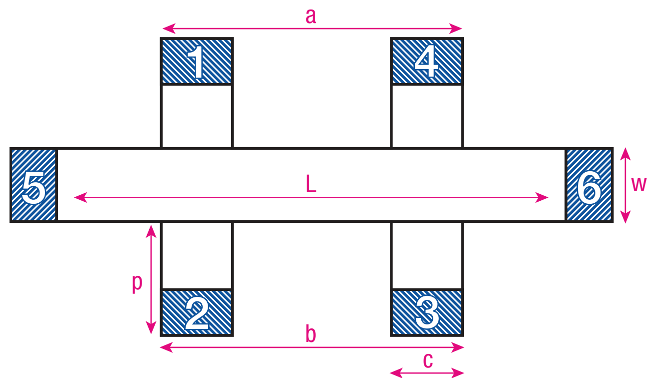


Figure 2 *A common Hall bar structure. These structures have a thickness, t, and the darkened areas show the locations for the electrical contacts.*

The test sample must have a method that allows the test sample to connect with instrumentation to make the desired characterization measurements, in this case Hall measurements. Connections need to be made to the sample; and, each connection creates a contact resistance. Keeping the contact resistance as low as possible and ohmic will reduce the impact of the contact resistance on the measurements. Ohmic refers to a contact that is linear and obeys Ohm's Law. A metal-semiconductor connection or interface has Schottky diode characteristics, which makes obtaining an ohmic contact a difficult challenge. Low resistance of the ohmic contacts compared with the level of the resistance of the test sample creates the least amount of measurement error by making the non-linear components of the contact insignificant. A high contact resistance leads to increased self-heating of the sample and creates a thermal error voltage. For a fixed excitation current, a low contact resistance enables a higher voltage measurement across the sample. Figure 3 shows a model for a metal-semiconductor contact.

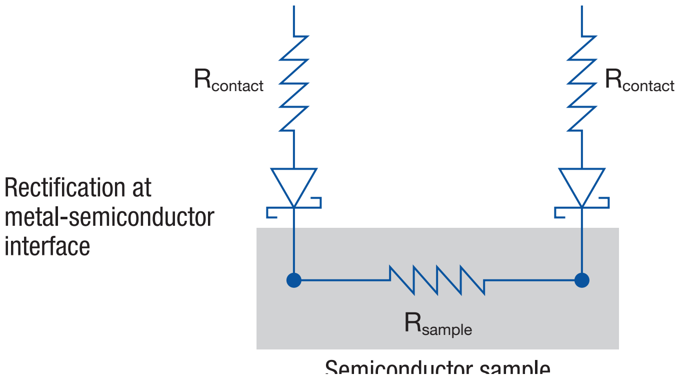


Figure 3 *A model for a metal-semiconductor contact showing a resistance element and a series Schottky diode.*

There are several methods for contact deposition including:

- Applying metal-based paints and pastes
- Melting metals directly on the semiconductor surface
- Evaporation sputtering
- Molecular beam epitaxy
- Ion implantation

Once the contact material has been deposited on the semiconductor, the contact can be thermally annealed by conventional oven, laser, an electron beam, or a rapid thermal annealing process that uses halogen lamps to quickly heat the semiconductor to the annealing temperature and hold the temperature for 10 to 30 s.

Whichever deposition technique is used, attention to detail and good techniques will minimize non-ohmic effects. Four important considerations for ensuring the quality of sample contacts include the choice of connection technique, contact material, contact placement and sample construction, and the stability of the connection.

1. **Connection method:** A contact can be either a metal pad with a wire bonded to the pad or a metal pad on which a probe needle can rest. See Figure 4. The choice of connection method will depend on both the type of sample and accessibility to the sample test fixture. Pads with bond wires are more suitable for samples housed in a cryogenic chamber for low temperature. Some samples or situations may preclude the use of contacts layered on the material; in those cases, probe needle contact methods may be required.

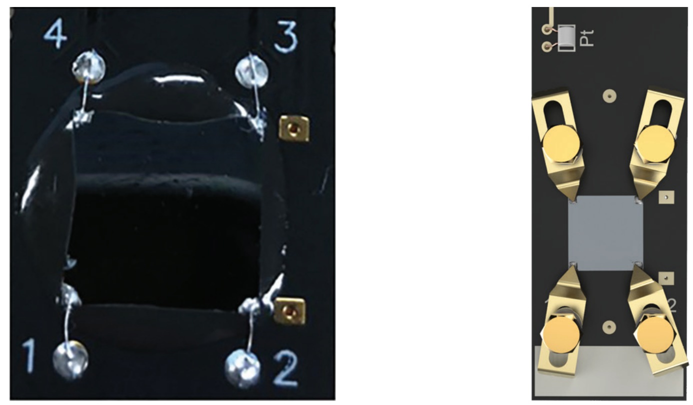


Figure 4 *A square van der Pauw structure showing wire bonding connections to an InAs sample (left) and probe connections to a bare sample card (right).*

2. **Contact material:** Use of high conductivity materials is recommended for metal pads. The details of the pad construction depend on the material. For example, for a p-type silicon sample, aluminum is the recommended contact material. For gallium oxide (Ga_2O_3), the preferred material is titanium/gold (Ti/Au), or titanium/aluminum (Ti/Al). Options for attaching wires to the contacts include conductive paint, indium solder, or silver epoxy. The determination is a factor of how well the material adheres to the sample.

	Recommended contact material	Contact resistance
Si p-type	Al-Si alloy	15 Ω μm^2
Si n-type	Al-Si alloy	90 Ω μm^2
SiC	Ti	0.0092 Ω cm^2
Ga_2O_3	Ti/Au	0.0002 Ω cm^2
n-GaAs	AuGe/Ni	
p-GaAs	Au/Pd	
n-InP	Au/Ti	

Table 1 *Recommended contact materials and their contact resistance for a set of semiconductor samples.*

3. **Contact placement and sample construction:** Ensure that metal pads are located in appropriate locations as defined by the van der Pauw or Hall bar structures. Also make sure pad size is small so that the ratio between the contact width, w, and the length, L, of the sample structure exceeds 1:10.

Geometrical error sources in a Hall bar, shown in Figure 5, are the result of deviations of the actual measurement geometry from the ideal of a rectangular solid with constant current density and point voltage contacts. If the aspect ratio of sample length to width, $L/w = 3$, then the deviation error from the ideal is less than 1%. Ensure that sample dimensions have $L/w \geq 3$. The finite size of the contacts affects both the current density and the electric potential in the vicinity of the contacts. The contact size can lead to relatively large errors. The errors are larger for a simple rectangular Hall bar than for one in which the contacts are placed at the end of the arms.

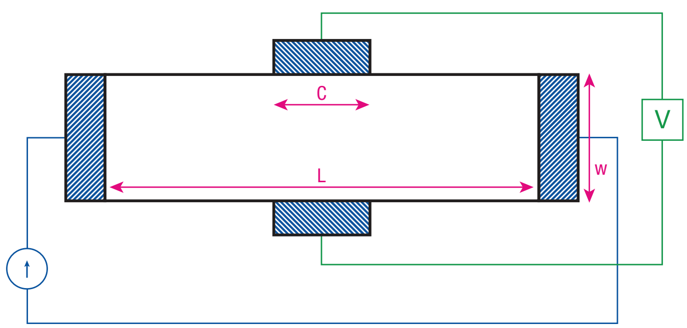


Figure 5 *A Hall bar structure with finite size voltage contacts.*

For a simple rectangular Hall bar, the error in the Hall mobility, μ, can be approximated (when $\mu B \ll 1$) by:

$$\frac{\Delta\mu_H}{\mu_H} = 1 - \left[(1 - e^{\frac{-\pi L}{2w}})(1 - \frac{2c}{\pi w})\right]$$

In this formula, the amount, $\Delta\mu_H$, is the amount that μ_H must increase to obtain a correct value. As an example, if $L/w = 3$ and $c/w = 2$, then $\Delta\mu_H / \mu_H = 0.13$ or 13%. This is a significant error.

Placing contacts at the ends of the contact arms (Figure 6) reduces the contact size error to tolerable levels. If $L/w \geq 4$, $c/w \leq 1/3$, and $p \approx c$, the deviations are small.

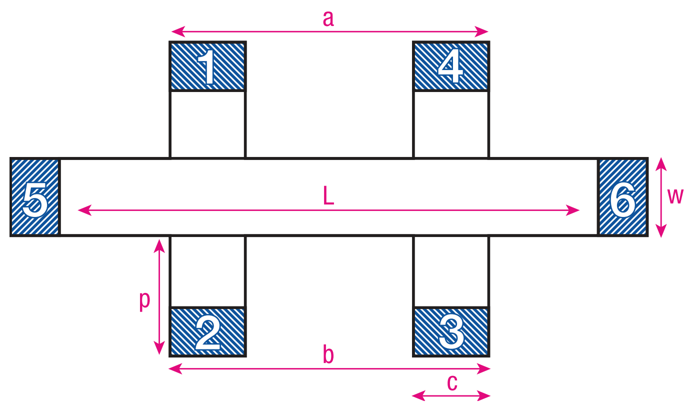


Figure 6 *A Hall bar with contact arms.*

Geometric errors in van der Pauw samples are the result of methods used to derive resistivity in which point-like contacts are assumed. For small sample structures, achieving point-like contacts is an unlikely scenario.

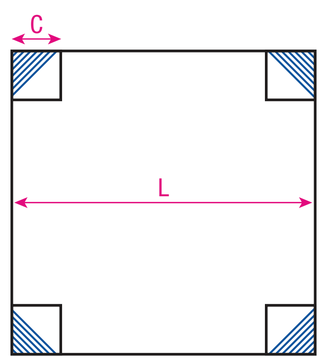


Figure 7 *A square van der Pauw structure with square or triangular contacts.*

The resistivity correction factor for a square van der Pauw structure is roughly proportional to $(c/L)^2$ for both square and triangular contacts. When $c/L = 1/6$, $\Delta\rho/\rho = 2\%$ for four identical square contacts and $\Delta\rho/\rho < 1\%$ for four identical triangular contacts. The Hall voltage measurement error is higher. It is directly proportional to c/L and is about 15% for triangular contacts when $c/L = 1/6$.

Circular van der Pauw structures shown in Figure 8 offer a little less error than square structures.

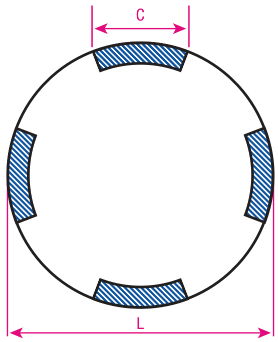


Figure 8 A circular van der Pauw structure.

The resistivity correction factor for contacts on a circular structure is:

$$\frac{\Delta\rho}{\rho} = -\frac{1}{16\ln 2}\left(\frac{c}{L}\right)^2 \text{ (per contact)}$$

This results in a correction factor of $\Delta\rho/\rho = -1\%$ when $c/L = 1/6$. The error for the Hall coefficient is:

$$\frac{\Delta R_H}{R_H} = \frac{2c}{\pi^2 L} \text{ (per contact)}$$

When $c/L = 1/6$ for all four contacts, the Hall coefficient error is 13%.

By creating slots to make the structure a cloverleaf as shown in Figure 9, the resistivity error can be reduced by a factor of 10 to 20. The slots also reduce the error in the Hall coefficient term by a factor of 3 to 5. The slots mechanically weaken the structure unless it is mounted on a thick substrate. Also the active area of the cloverleaf is much smaller than the complete sample.

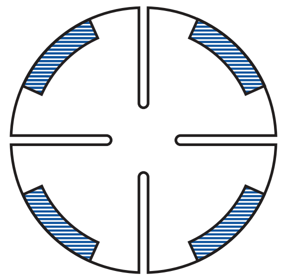


Figure 9 A cloverleaf van der Pauw structure.

The Greek cross structure, Figure 10, is a superior van der Pauw form factor for minimizing the error created by contact size on the structure. Its advantage over the square and circular van der Pauw structures is analogous to having Hall bar contacts at the ends of arms.

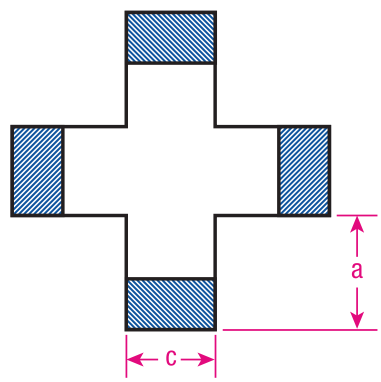


Figure 10 A Greek cross van der Pauw structure.

The error term for the deviation of the measured resistivity, ρ_m, from the actual resistivity, ρ is:

$$E = 1 - \frac{\rho}{\rho_m} = (0.59 \pm 0.0006)\, e^{[-(6.23 \pm 0.002)\frac{a}{c}]}$$

When the ratio of the contact arm to the total length of the structure, $c/(c + 2a)$, is 1/6, then E is approximately 10^{-7}. In addition, the Hall mobility error is:

$$\frac{\mu_H - \mu_{Hm}}{\mu_H} = \frac{\Delta\mu_H}{\mu_H} = 1.045 e^{-\pi a/c} \text{ (four contacts)}$$

in which μ_H and μ_{Hm} are the actual and measured Hall mobilities, respectively. When $c/(c + 2a) = 1/6$, then $\Delta\mu_H/\mu_H \approx 0.04\%$. The trade-off for low error is a more complex structure that is the most difficult of the above structures to fabricate.

4. **Stability of the connection:** Bonded wires lack flexibility but produce constant forces on the sample. Probe contacts can have variable pressure on the sample, particularly in variable temperature measurements (due to thermal expansion and contraction); and the probe contacts are susceptible to sources of vibration. These effects can cause variations in contact resistance, which can create measurement error. Ensuring that each probe applies the same pressure to the sample eliminates a variation in contact resistance among the multiple probes.

The selection of contact material, how well it is bonded to the sample, and its size and location contribute to measurement error. Of greater significance is the non-linearity, the non-ohmic characteristic of the metal contact-to-semiconductor interface.

Misalignment voltage error

A second significant geometric error term is known as the misalignment voltage, V_m. The misalignment voltage is the measured Hall voltage with no magnetic field applied to the sample. This error term is, in most cases, the largest intrinsic error in a Hall measurement. Using a square van der Pauw sample as an example structure, the misalignment voltage would be 0 V if the sample structure was a perfect square and the contacts were theoretical point contacts located on the corners of the structure. Any deviation from the ideal geometry results in a non-zero misalignment voltage. Thus the misalignment voltage is a factor in all Hall measurements.

Figures 11 to 13 illustrate the magnitude of the misalignment voltage for a square van der Pauw structure. Figure 11 shows the equipotential lines and current flow for the ideal geometry, a perfect square with point contacts. The ideal geometry does not generate a misalignment voltage. Note that in this particular example, the sample has a sheet resistivity of 1000 Ω/square and the mobility is a low 1 cm^2/V s. The Hall voltage is only 100 µV when the applied magnetic field, B, is 1 T and the source current is 1 mA. If the contacts do not have identical geometries as shown in Figure 12, the equipotential lines become non-uniform and the zero magnetic field misalignment voltage reaches 66 mV with a 1 mA source current. In this case, the misalignment voltage is 600 times greater than the Hall voltage. Figure 13 shows the non-uniform equipotential lines and current flow when the dimensions of the sample are not exactly square. In this case, the misalignment voltage becomes 127 mV, over 1000 times larger than the Hall voltage.

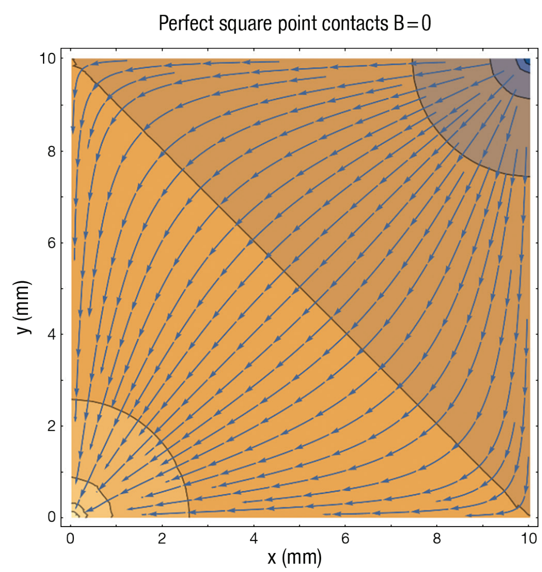


Figure 11 *Equipotential lines and current flow for a square sample with ideal point contacts. The misalignment voltage is 0 V.*

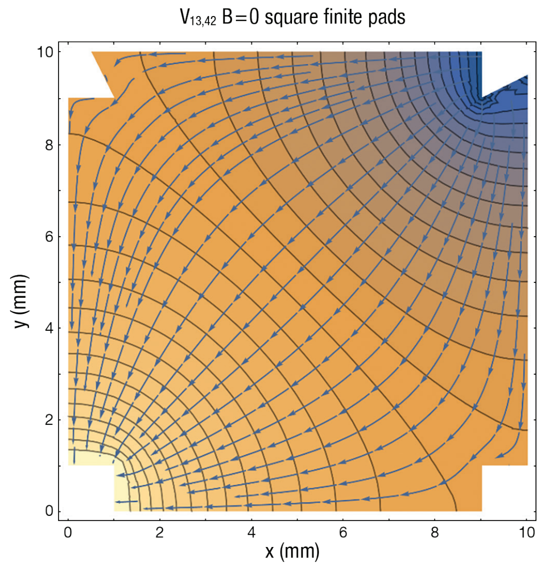


Figure 12 *The square sample with finite deformed contacts. The equipotential lines are no longer uniform. This non-uniformity results in a misalignment voltage of 66 mV.*

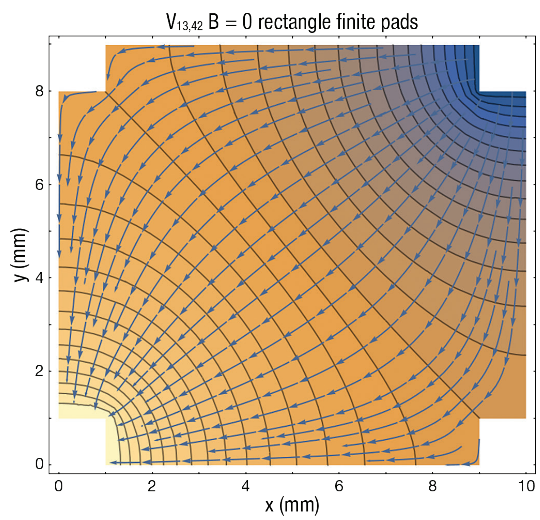


Figure 13 *The sample is no longer a square with width/length = 0.9; but the contacts are symmetric and identical. The misalignment voltage is 127 mV.*

The misalignment voltage can become decades larger than the Hall voltage. With a sample material that has high resistivity and low carrier mobility, special techniques are needed to extract the Hall voltage from the large misalignment error voltage.

Mitigating the effects of the contact resistance errors and the geometric errors on Hall measurement accuracy

While the sources of error pose significant challenges for making accurate Hall effect measurements, there are methods to minimize their impact on the measurements. The following sections offer solutions for minimizing and eliminating these errors.

Minimize the effect of non-ohmic contacts

Making current-voltage *(I-V)* measurements on the contact pairs allows analysis of the ohmic quality of the contacts. The more linear the *I-V* pairs, the more ohmic are the contact-sample interfaces. An excellent method to assess the ohmic quality of the contact is to perform a correlation of the measured *I-V* data with an ideal linear function using a regression fit methodology. Using a correlation coefficient of greater than or equal to 0.9999 as a threshold for a good ohmic contact ensures that the non-ohmic contact-sample interface will have a negligible impact on the resistivity and Hall voltage measurements.

The Lake Shore MeasureReady™ M91 FastHall™ measurement controller automatically performs both *I-V* measurements on pairs of contacts and performs the correlation of the data with a regression line. To obtain the 0.9999 or higher correlation, the M91 adjusts the magnitude of the source current so that the metal-semiconductor contact diode is operated in its linear region as shown in Figure 14. The process of determining a source current that results in a correlation coefficient of 0.9999 or greater is an iterative process. The trade-off is between a low source current to maximize the correlation coefficient and a high enough current to achieve a reasonable signal-noise ratio (SNR) for the Hall voltage measurement. The M91 starts with its highest source current and progresses to lower values until a correlation coefficient of 0.9999 is attained. The optimum solution is the largest current that yields a good correlation and a measurable Hall voltage that yields consistent values and a consistent polarity over multiple measurements. Figures 15 and 16 show examples of non-ohmic contacts and linear ohmic contacts.

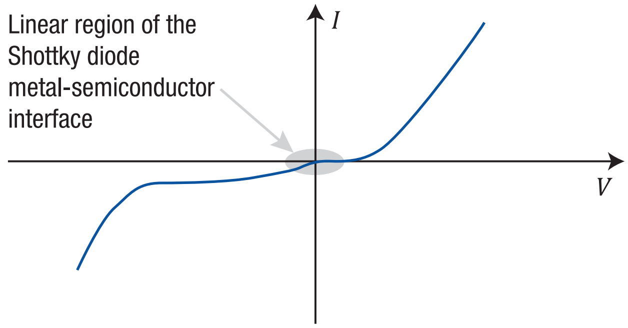


Figure 14 *The linear region of a Schottky diode representing the metal-semiconductor interface.*

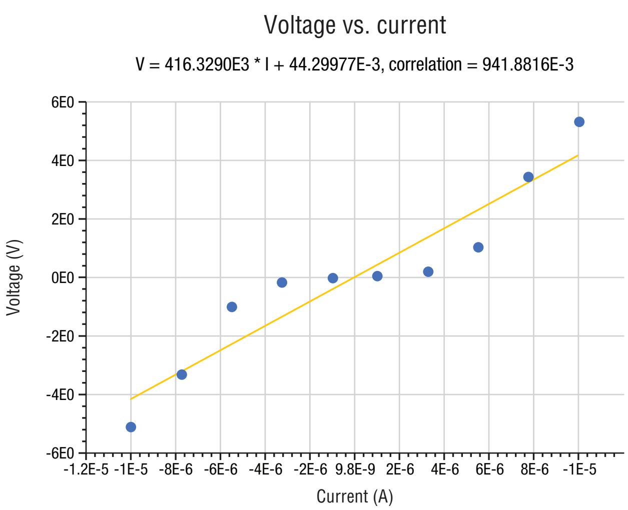


Figure 15 *I-V curve showing a non-ohmic, Schottky diode-like characteristic compared with the best fit straight line.*

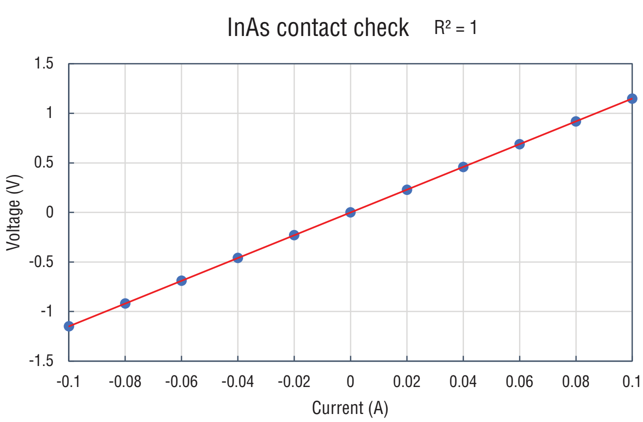


Figure 16 *I-V curve showing an excellent ohmic contact.*

Minimizing the effect of the misalignment voltage

The Hall voltage is proportional to the magnetic field intensity, B, source current, I, the Hall coefficient, R_H, and inversely proportional to the sample thickness, t. As previously stated, a structure with an ideal geometry has a measured Hall voltage of 0 V in the absence of an applied magnetic field.

With a real sample, the measured Hall voltage, $V_{measured}$, will include a misalignment error term, V_m, and a thermoelectric voltage, V_{TE}. The misalignment voltage is proportional to the resistivity of the material, ρ, the source current, I, and a factor, α, that depends on the sample geometry. The factor converts resistivity of the sample to resistance between the two Hall voltage contacts. The third term is a thermoelectric term which results from contacts between two different materials; and, the thermoelectric voltage is independent of the source current. Also included in the thermoelectric voltage are voltages caused by any thermal gradients in the sample.

$$V_{measured} = \frac{R_H IB}{t} + V_m + V_{TE}$$

$$V_{measured} = \frac{R_H IB}{t} + \alpha\frac{\rho}{t}I + V_{TE}$$

The mobility, μ, is the Hall coefficient, R_H, divided by the resistivity, ρ. Then using $R_H = \mu\rho$, $V_{measured}$ can be re-stated as:

$$V_{measured} = \frac{\rho I}{t}(\mu B + \alpha) + V_{TE}$$

The factor, α, can be as low as 0, representing no offset, but it is typically around 1.

Since the misalignment voltage is not dependent on the magnetic field, the misalignment voltage can be canceled by repeating the measurement with the magnetic field reversed and subtracting the two voltage measurements.

One problem is that when the Hall voltage is small due to low carrier mobility, the subtraction of two larger voltages will produce a small result. Any noise in the measurement can contribute to inaccurate results, including a change in polarity of the Hall voltage representing a different dominant charge carrier in the sample. Extracting a small voltage in the presence of a large error voltage is a disadvantage of using a DC field and a DC measurement of the Hall voltage. A superior alternative is an AC technique, in which a sinusoidal magnetic field is applied to the sample and a phase-sensitive detection method is used to measure the small Hall voltage. The problem with both these methods, particularly the AC method, is the time needed to make the measurement. The DC method requires the reversal of the magnetic field polarity, and the AC method requires a very low frequency, around 0.2 Hz, to drive the large inductance of the magnet and create an AC field. Long measurement time intervals can result in changes in the temperature of the sample, which results in differential changes in misalignment voltage. This differential voltage as a function of time will have a frequency component identical to the frequency of the magnetic field. This error term cannot be removed from the Hall voltage measurement.

A solution is to use a new technique that combines extraction of small voltages in the presence of large error voltages and fast measurements to avoid sample thermal drift. The technique uses reciprocity methodology to eliminate changing magnet polarity — resulting in a faster measurement — and uses a bipolar square wave source current. The combination of reciprocity and bipolar current can eliminate the misalignment voltage and the thermoelectric voltage.

The main principle is based on the reciprocity theorem. For a linear, passive network with a current source creating a signal and a voltmeter performing a measurement, the measured voltage does not change if the position of the current source and voltmeter are interchanged. However, in the presence of an external magnetic field, the interchange of the current source and voltmeter does not result in an equivalent measurement unless the polarity of the magnetic field is reversed. Effectively, changing the position of the current source and the voltmeter in the presence of a magnetic field simulates a magnetic field reversal and results in a voltage measurement of the opposite polarity. The reciprocity theorem can apply to Hall measurement since most semiconductor samples are equivalent to linear, passive networks.

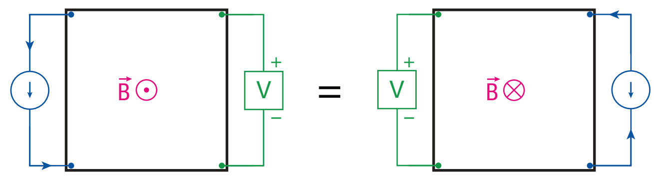


Figure 17 *The reverse field reciprocity theorem. In the presence of a magnetic field, interchanging the current source and the voltmeter has the effect of reversing the polarity of the magnetic field.*

The Lake Shore MeasureReady™ M91 FastHall™ measurement controller, using patented technology, combines the reciprocity theorem in the presence of a magnetic field to remove the unwanted misalignment voltage with a bipolar current source, which can eliminate the thermoelectric voltage. By using the principles of the reciprocity theorem, the M91 avoids the lengthy time interval needed to reverse the magnetic field polarity. The M91 also uses its bipolar current source to change polarity out of phase with the reversal of the positions of the current source and the voltage measurement instrument. This enables a phase sensitive detector to extract a small AC-like voltage in the presence of a large DC offset due to the misalignment voltage and a thermoelectric voltage.

By eliminating the time required to reverse magnet polarity and aided by fast solid state switching that reverses the positions of the current source and the voltage measurement circuitry, the M91 performs Hall measurements in millisecond time intervals. Hall measurements taken at that speed eliminate any changes in temperature gradients across the sample. The short time interval avoids a differential misalignment voltage due to a temperature gradient and a thermoelectric voltage gradient. Neither differential error voltage can be removed by either field reversal or bipolar current sourcing methods.

Figure 18 shows the four steps used by the M91 to eliminate the misalignment voltage error and the thermoelectric voltage error terms.

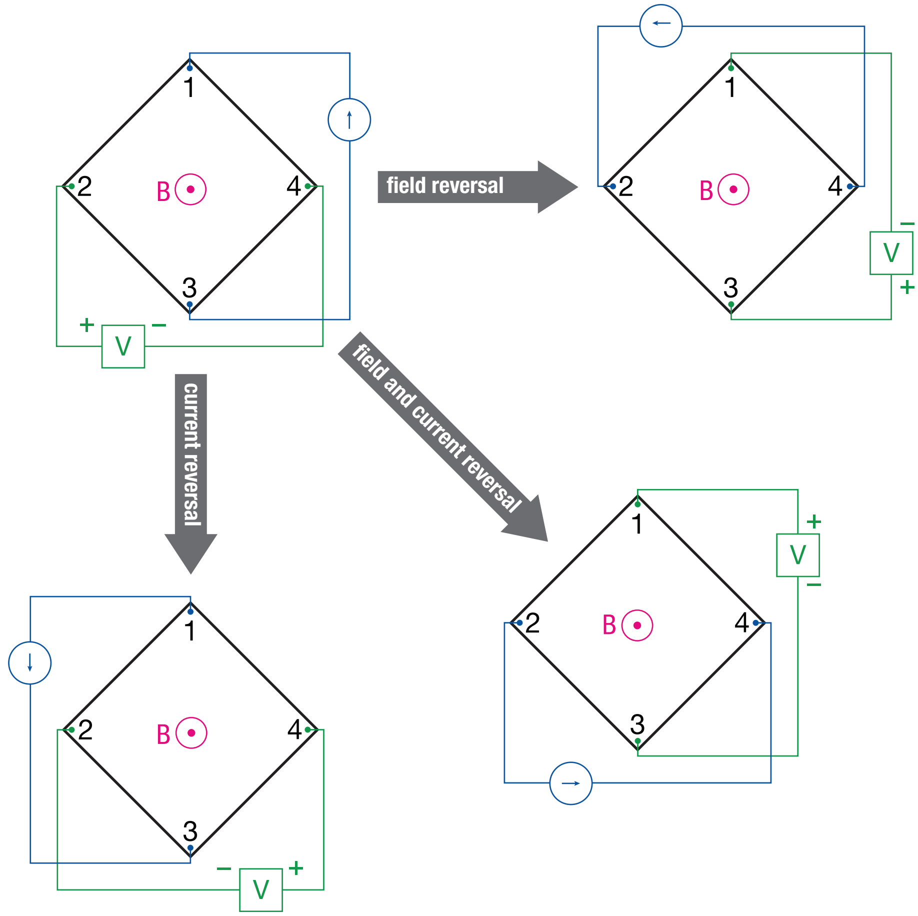


Figure 18 *The M91 FastHall measurement controller uses both reciprocity and bipolar current sourcing to perform fast Hall measurements and eliminate the misalignment voltage and the thermoelectric voltage error terms.*

Figure 19 *The M91 FastHall measurement controller*

Summary

A Hall effect measurement is a challenging measurement subject due to a number of error sources. The geometry of the sample alone has limitations on Hall measurement accuracy. However, we recommend that you use a geometry that can easily be made with consistent dimensions and repeatable metal contact dimensions. Also ensure that the sample is not too fragile so that deformation and breakage is avoided. A square van der Pauw geometry is an easily fabricated sample that can satisfy these recommendations. Pay attention to good assembly technique and proper materials to create low metal-to-semiconductor contact resistance. Characterize the ohmic quality of the sample contacts. Use magnetic field reversal or an AC field and current reversal to eliminate misalignment voltage and thermoelectric voltages from the Hall measurement.

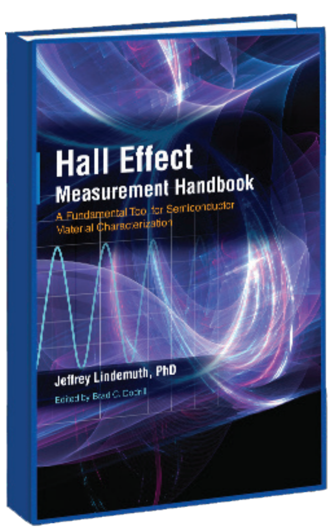
With van der Pauw samples, The M91 can simplify and speed up the task of obtaining quality Hall effect measurements. The M91 automatically:

- Characterizes the ohmic quality of the sample contacts
- Determines the optimum settings for sourcing and measuring Hall parameters
- Performs fast measurements to cancel out fixed misalignment errors and avoid differential misalignment errors by eliminating the need for either magnetic field polarity reversal or a sinusoidal magnetic field

The FastHall technique is particularly valuable for low mobility materials which require Hall measurements that can take minutes to an hour. As a result, the M91 can determine mobilities as small as 0.001 cm^2/V s. With all the challenges of Hall effect measurements, the M91 can automatically provide quality results in the shortest time.

For more information on Hall effect theory, measurements, error sources, and primary reference sources, refer to the Hall Effect Measurement Handbook written by Dr. Jeffrey Lindemuth, Lake Shore Senior Staff Scientist.

Get your free copy at www.lakeshore.com/HHB



To learn more about Hall measurements and the advantages of the FastHall™ measurement technique, watch the webinar given by Dr. Lindemuth and Lake Shore Engineering Applications Director Kevin Carmichael.

Watch the webinar at www.lakeshore.com/FHW

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