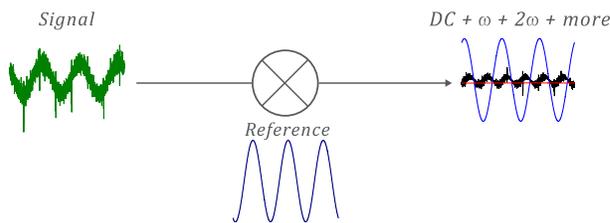


# Hybrid vs. FIR and IIR Filtering for Faster Lock-In Measurements



**Figure 1** Lock-in product detector. The input stage of a lock-in measures a voltage signal that contains an AC modulated signal (at frequency  $\omega$ ), a DC offset, noise, and AC signals at other frequencies. The DC component of the post-mixed signal contains information about the signal at frequency  $\omega$  while other components need to be removed by the output filter.

## How do you measure AC signals when the noise is larger than the signal?

Lock-in amplifiers can measure AC signals in the presence of noise much larger in magnitude than the signal. Lock-ins achieve this high degree of sensitivity by mixing the measured signal with a single-tone AC reference signal (Figure 1). The output of the mixing results in a DC voltage of which the magnitude depends on the size of the signal at the reference frequency, but also includes other AC voltages (notably at the reference frequency) that need to be removed. To isolate the DC voltage (the desired signal), these higher-frequency components are removed with the so-called output stage filter.

Traditionally, a low-pass recursive filter (RC filter), also known as an infinite impulse response (IIR) filter, has been used as the output stage filter. For suitable rejection of these higher-frequency components, the IIR filters require long wait times for settled values. In this application brief, we will introduce an alternative filter, the moving average filter, or finite impulse response (FIR) filter, and demonstrate the measurement advantages of a hybrid IIR + FIR filter over the traditional IIR filter.

### The IIR filter quandary

As the name implies, the output of an IIR filter approaches the final value exponentially, and like Achilles and the tortoise, never completely reaches the final value. For instance, a 12 dB roll-off filter with a 1 s time constant takes 6.6 s to settle to a value that is 99% of the final value. Higher-value roll-off filters and longer time constants are used in a lock-in measurement to improve rejection of the harmonics in the output stage and lower the noise of the measurement; however, this improvement in signal quality comes with a substantial increase in settle time and a slower lock-in acquisition (Table 1).

IIR filter roll-off	6 dB	12 dB	18 dB	24 dB
99% settle time	4.6 $\tau$	6.6 $\tau$	8.4 $\tau$	10.1 $\tau$

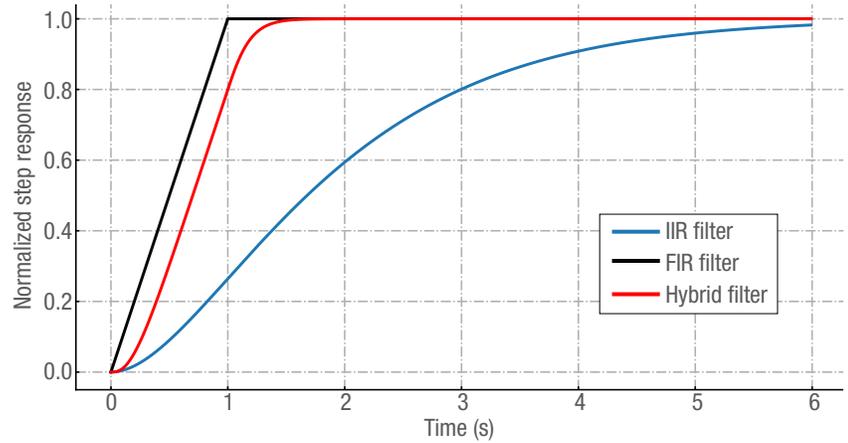
**Table 1** Time for a IIR filter to settle to 99% of final value for a chosen time constant,  $\tau$ .

## The FIR filter – pros and cons

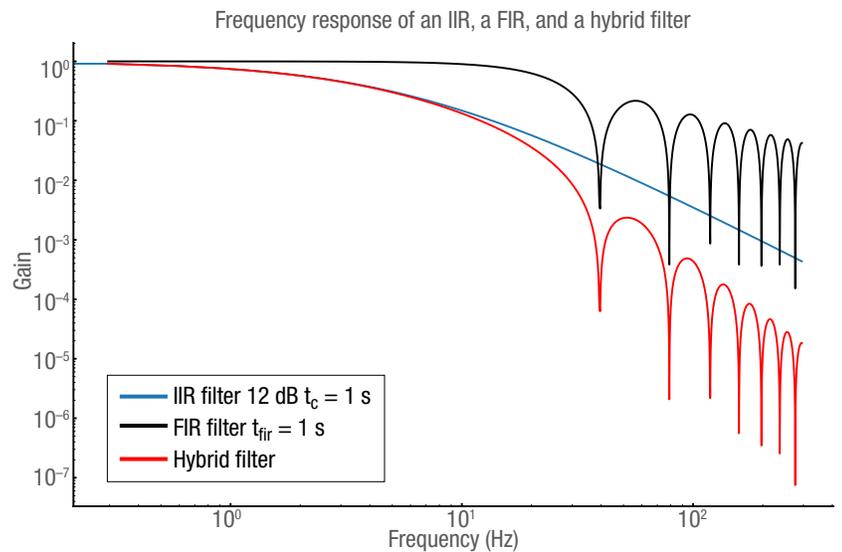
The FIR filter implementation uses a moving average with a user-selected averaging time. In Lake Shore's M81-SSM synchronous source measure system, the FIR filter time is set in reference frequency cycles; for example, with a 10 Hz reference frequency, a 1 s averaging time is 10 cycles while a 2 s averaging time is 20 cycles. The key advantage of the FIR filter is that the measurement settles to exactly 100% of the final value in the averaging time (Figure 2). For the same noise reduction of the output filter, the FIR filter will always settle faster than a IIR filter, which often results in significant reduction in overall measurement time. A second advantage of the FIR filter is that when the filter time is set to an integer number of cycles, the harmonics of the reference frequency in the output of the product detector, as shown in Figure 1, are exactly canceled. Although the FIR filter is very good at rejecting harmonics of the reference frequency, spurious signals at other frequencies, like line frequency and harmonics, are not attenuated as well as an IIR filter. In Figure 3, the black line is the frequency response of a FIR filter. The filter has notches at any frequency that is a harmonic of the FIR filter time. Between two consecutive notches, there is a relative maximum in the response function. The blue line is the frequency response of an IIR filter. It is a smooth decreasing function of frequency but does not suppress the harmonics as well as the FIR filter. The green line is the frequency response of the hybrid filter. The notches from the FIR are still there, but the relative maximums have been decreased to below the IIR filter.

## The hybrid lock-in filter

The combination of IIR and FIR filters into a hybrid lock-in filter enables lock-in users to achieve equivalent signal to noise ratios and good spurious signal rejection — all with faster measurement times. In the M81-SSM, the hybrid filter is digitally implemented with an IIR filter, with time constant  $\tau$ , followed by an FIR filter with settle time  $t_{fir}$ . Different ratios of  $\tau/t_{fir}$  allow the experimentalist to tune the relative amount of FIR and IIR filter capability for a given application. In this brief, we will focus on a general-purpose hybrid filter with the ratio  $\tau/t_{fir} = 0.1$ . Table 2 shows the settling time of this hybrid filter for a 99% settle time criteria. The hybrid filter provides faster settle times for the same noise reduction as an IIR.



**Figure 2** Step response of three filters. The IIR filter is a 12 dB slope 1 s time constant. The FIR filter (black) is a 1 s filter time. The hybrid filter is a 1 s FIR time and a 12 dB IIR filter with time constant 0.1 s.



**Figure 3** Frequency response of an IIR filter, an FIR filter, and a hybrid filter. Notice that at the harmonics of the FIR filter time, the response is zero. The FIR filter can have a higher response (less rejection) than the IIR filter; whereas, the hybrid filter (product of the IIR and FIR) is always lower than an IIR filter with equivalent bandwidth.

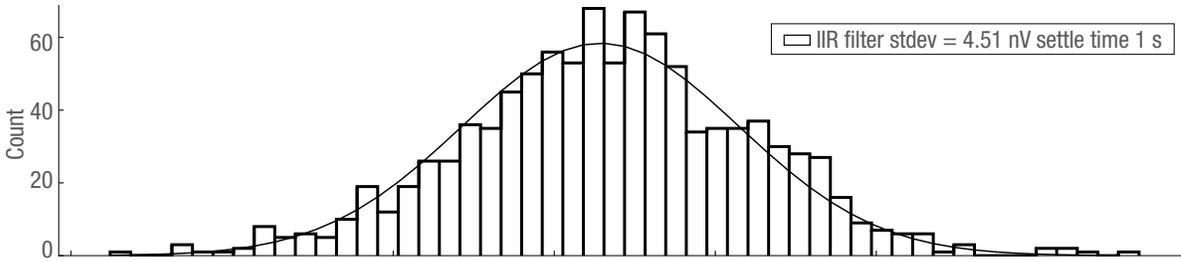


Settle time hybrid filter with $\tau/t_{fir} = 0.1$				
IIR filter roll-off	6 dB	12 dB	18 dB	24 dB
99% settle time	$1.20 t_{fir}$	$1.41 t_{fir}$	$1.57 t_{fir}$	$1.73 t_{fir}$

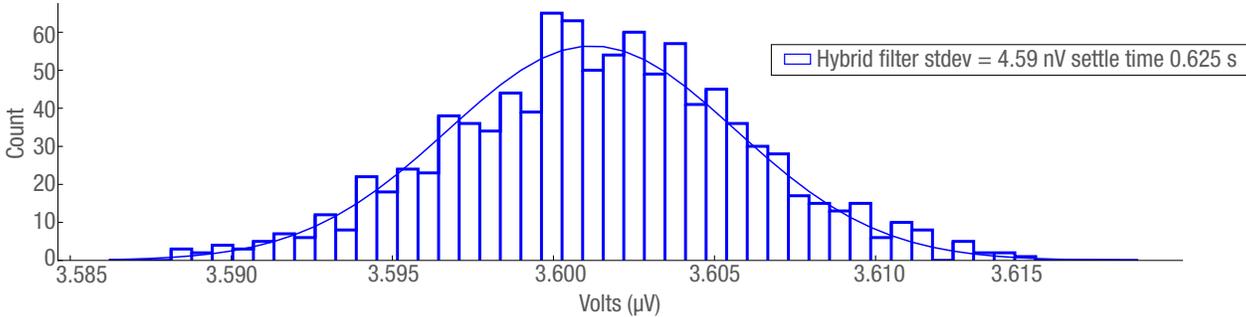
**Table 2** Time for a hybrid filter to settle to 99% of final value.

To use this chart, pick an IIR roll-off, say 12 dB, and the desired measurement time, say 1 s. This is the rate which you can record statistically independent data points. Using a 12 dB roll-off the value of  $t_{fir} = 1 \text{ s}/1.41 = 0.7092 \text{ s}$ . If the reference frequency is 20 Hz, this is 14.2 cycles of the reference frequency. Since we need an integer number of cycles for the FIR filter implementation, round this to 14 cycles. The FIR filter time is then 14 cycles/20 Hz = 0.7 s, and the IIR time constant is the  $0.1 \cdot 0.7 = 0.07 \text{ s}$ . The settle time with the rounded FIR filter times is  $1.41 \cdot 0.7 \text{ s} = 0.987 \text{ s}$ .

**Figure 4 [A]** 35 mΩ resistor noise histogram with FIR filter



**Figure 4 [B]** 35 mΩ resistor noise histogram with hybrid filter



**Figure 4 [A,B]** Two measurements of a 35 mΩ resistor with a M81-SSM BCS current source (BCS-10), and VM voltage measure (VM-10) module, in lock-in mode. The top figure [A] is a histogram of 1,000 measurements using an IIR filter. The measurement time was 1 s and the standard deviation is 4.51 nV. The bottom [B] histogram is the same measurement, but with a hybrid filter with settle time of 0.625 s. The standard deviation of the measurement is 4.59 nV, essentially the same as the IIR filter but 37% faster than the IIR filter.

Here the voltage noise from a 0.035 Ω resistor is used to compare the measurements of the traditional and hybrid lock-in filters. The resistor is measured with an M81-SSM using the BCS-10, a balanced current source, and a VM-10 in lock-in mode. The BCS-10 current is 100 µA RMS at 83 Hz. The top-most plot in Figure 3 shows the distribution of lock-in voltage measurements using an IIR-only filter with 12 dB/oct roll-off and time constant = 0.108 s, which is 0.1% settled after 1 s. The data was gathered at 1 measurement per second for 1,000 measurements. The standard deviation for the IIR filter is 4.51 nV. The measurement distribution for the hybrid filter is shown in the bottom of Figure 4. The fast hybrid filter consists of a 31 cycle FIR filter and an IIR time constant of 0.037 s. With a 4.59 nV standard deviation from 1,000 measurements, the hybrid filter shows effectively identical noise as a 12 dB IIR-only filter but with a settling time of 0.625 s compared to the 1 s of a IIR-only filter.



## The Allan Deviation

In a system dominated by white noise, the standard deviation of measurements should decrease proportionally to the square root of the settle time. This relationship can be visualized using an Allan deviation plot, which displays the standard deviation against the settle time on a log-log scale. Ideally, the plot should exhibit a straight line with a slope of  $-0.5$ . To demonstrate this behavior, the variation in a resistance measurement of a  $50\ \mu\Omega$  shunt (Ohmite SHA series, 1% accuracy, 100 PPM/ $^{\circ}\text{C}$  temperature coefficient at  $20^{\circ}\text{C}$ ) was measured as a function of measurement time. An M81-SSM outfitted with a BCS-10 sources 100 mA peak amplitude current at 30 Hz to the resistor, while a VM-10, configured for lock-in detection, measures the voltage across the shunt. The lock-in was set-up using a hybrid filter with settle times ranging from 100 ms to 100 s/pt. To maintain the statistical independence of the lock-in data, each data set waited a full settle time between each of the 1,000 resistance measurements.

Figure 5 presents the standard deviation of 1,000 resistance measurements at various settle times using the M81-SSM configuration, and a fit to the data shows the predicted  $-0.5$  slope in the standard deviation plot. While it appears that with longer and longer averaging times, this measurement could reach single n $\Omega$  noise levels, environmental factors ultimately will limit the noise floor of the measurement. As the settle time of the lock-in filter increases, environmental fluctuations such as temperature, mechanical noise, water flowing, and others can introduce low-frequency noise ( $1/f$  noise) into the overall measurement system. This causes the standard deviation to deviate from the expected square root relationship, eventually exceeding the predicted values. In this  $1/f$  noise-dominant region, increasing the settle time provides little or no improvement in measurement precision. It is therefore crucial to experimentally determine the maximum settle time for a given measurement setup. This involves analyzing the Allan deviation plot (like the one shown in Figure 5) and identifying the inflection point where the curve starts to deviate from the ideal  $-0.5$  slope. This point indicates the optimal settle time, beyond which further increases in filter time offer diminishing returns in reducing measurement uncertainty.

Allan deviation plot  $50\ \mu\Omega$  resistor with BCS-10 VM-10 modules

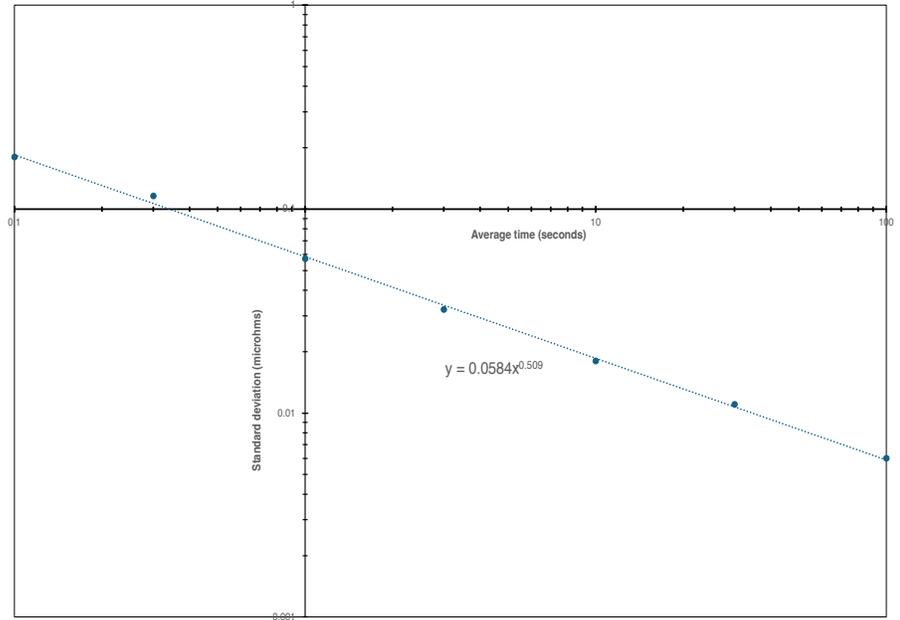


Figure 5 Allan deviation of  $50\ \mu\Omega$  resistor.

## Conclusion

The output filter is a key component of lock-in detection. Using the M81-SSM FIR output filter in addition to the IIR filter often allows for faster measurements at the same signal to noise ratios compared to the IIR filter. In circumstances where the measured signal contains large interfering signals or signals in close proximity to the reference frequency, the hybrid filter has better noise rejection compared to the IIR filter. The M81-SSM allows users to easily switch between FIR, IIR, and hybrid FIR/IIR output filtering.

