Finite sample size effects on the calibration of Vibrating Sample Magnetometer

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Abstract– The normal procedure for calibration of the moment readings of a VSM is to use a Ni sphere of known magnetic moment to determine the moment/volt of the VSM sensing coils as measured by a lockin amplifier. The assumption is that when an unknown sample is placed in the VSM the same moment/volt will be produced. If the sample is a thin film and is large compared to the size of the Ni sphere errors as large as 20% can be introduced into the measurement. The field from a uniform magnetized sphere is a dipole field. Large diameter thin films cannot be approximated as point dipoles. In this paper an approximation for a thin film is used to calculate the signal induced in the VSM coils. The approximation is to treat the thin film as an oblate spheroid. This approximation has the advantage that the sample can be uniformly magnetized and solutions for the external field can be simply expressed in oblate spheroidal harmonics.

***Index Terms*—Magnetometers, Materials testing, Units and standards in magnetism.**

I. INTRODUCTION

VIBRATING sample magnetometers (VSM) have traditionally been calibrated by using a small high purity nickel sphere as the sample. The magnetic moment of the sample at saturation is known from the mass of the sphere and the voltage induced in the coils of the VSM is measured. This gives a calibration constant of the VSM as so many Am^2/V. The nickel sample is replaced by the sample to be measured and the calibration constant is used to convert voltage readings to a moment reading. Since the introduction of the VSM [1], various authors [2-5] have presented methods for calculating the voltage induced in different VSM coil configurations. These methods have predominantly used a point dipole approximation for the sample.

Current application of VSM measurements often requires the measurements of thin film samples with very small magnetizations. In order to increase the signals, the coils are often situated very close to the edges of the disk. In this case the approximation of a point dipole source is no longer valid. Ideally, the VSM should be calibrated with a known standard sample the same size and shape as the sample to be measured. The effect of a finite size sample is treated in this paper by the following procedure. First the magnetostatic problem for the sample geometry is solved. The thin film samples studied here will be approximated by an oblate spheroid. This geometry can be uniformly magnetized and the external field can be simply expressed in oblate spheroidal co-ordinates. Second the sample is assumed to sinusoidally vibrate in the vertical direction. The spatially varying time change of the flux through the sensing coils is integrated over the face of the coils to yield the voltage measured by the VSM. The voltage is compared to the voltage induced in the coils by a point dipole. The moment of the point dipole is normalized so that when the thin film is very far from the coils the voltage from the thin film and the voltage from the point dipole are the same.

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II. THEORY FOR FINITE SIZE SAMPLES

A. *Voltage Induced in a Coil from a Vibrating Sample*

A sample that has a magnetization $\vec{M}(x,y,z)$ will produce a field external to the sample (assumed to be free space) $\vec{B}(x,y,z) = \propto_0 \vec{H}(x,y,z)$. The $\vec{H}$ field is calculated from:

$$. \bullet \vec{H} = -. \bullet \vec{M} \, . \tag{1}$$

Recall, for a uniformly magnetized sample, $. \bullet \vec{M} = 0$, everywhere except on the surface of the sample. If the sample vibrates in the z direction with $z(t) = A\sin(. \ t)$, the B field at the observation point x,y,z will be $\vec{B}(x,y,z(t))$. If the center of a single turn coil of radius $.$ is located at (x_c, y_c, z_c) and its normal is in the x direction, the flux through the coils is:

$$\Phi(t) = \iint_{coil} B_x(x_c, y, z(t))dydz \, . \tag{2}$$

The voltage induced in the coil will be:

$$V \, . \, \frac{d\Phi}{dt} = A . \cos(\ t) \iint_{coil} \frac{dB_x(x_c,y,z)}{dz} dydz \tag{3}$$

We arrive at the well-known result that the signal induced in the coils of a VSM is proportional to the amplitude of vibration, the frequency of vibration and the gradient of the field from the sample in the direction of vibration. For the particular case of equation 3, the z integral can be easily solved:

$$\frac{d\Phi}{dt} = A \ \cos(\ t) \int_{y_c-.}^{y_c+.} . \, B_x(x_c, y, z(y))dy \tag{4}$$

Where for a given value of y, $\Delta B_x(x_c, y, z(y))$ is the change in B_x between the two z-values at the edge of the coil. This allows the calculation of the induced voltage without doing a field derivative. Equation 4 can then be integrated over center co-ordinates and coil radius to calculated the induced voltage from a multi-turn coil.

B. *Oblate Spheroidal Sample*

We will model a thin film disk of radius a and thickness $\delta, \delta/a \ll 1$ as an oblate spheroid of semi-major axis a and semi-minor axis a*η_0. The transformation for Cartesian co-ordinates x,y,z to oblate spheroidal (as used by Smythe[6]) ξ,η,ϕ is:

$$\begin{aligned} z &= a\eta\xi \\ x &= a((1+\eta^2)(1-\xi^2))^{1/2}\cos(\phi) \\ y &= a((1+\eta^2)(1-\xi^2))^{1/2}\sin(\phi) \end{aligned} \tag{5}$$

The surface of the oblate spheroid $\eta = \eta_0$ defines the disk. Here $0 \le \eta < \infty$ and $-1 \le \xi \le 1$. The magnetostatic potential from a uniformly magnetized oblate spheroidal is:

$$V = M_0 b\rho((\cot^{-1}(\eta) - \eta/(1+\eta^2))\cos(\varphi) \tag{6}$$

where

$$b = \frac{(1+\eta_0^2)}{((1+\eta_0^2)\cot^{-1}(\eta_0) - \eta_0)}, \tag{7}$$

M_0 is the magnetization and $\rho = \sqrt{x^2 + y^2}$. For a thin disk η_0, the ratio of the semi-minor axis to the semi-major axis, is small and $b \approx 2/\pi$.

To calculate the field at a point x,y,z, equation 5 is inverted to get ξ,η,ϕ.. The potential is calculated using equation 6 and the derivative of the potential are calculated by numerical differentiation using a four point approximation:

$$\frac{df(x)}{dx} \approx \frac{f(x+\Delta x) - f(x-\Delta x) + 2*(f(x+2\Delta x) - f(x-2\Delta x))}{10\Delta x} \tag{8}$$

Care must be taken in evaluating $\cot^{-1}(\eta) - \eta/(1+\eta^2)$ for large values of η due to loss of significance from the subtraction. For values of η > 100, a fifth order expansion of equation 6 in powers of $1/\eta^2$ was used.

C. *Results*

The procedure described in the previous section shows that the voltage induced in the VSM coils depends not only on the magnetic moment of the sample, but also its size and how far it is from the coils (the working gap of the VSM). To show this the induced voltage is calculated for different size disk and different coil configurations. Two different coil configurations were used in the calculations. Table 1 summarizes these coil configurations.

TABLE I
SUMMARY OF VSM COIL CONFIGURATION

	Inner diameter (mm)	Outer diameter (mm)	Length (mm)
Coil 1	5	23	9
Coil 2	3	11	5

Summary of the physical dimensions of the two coils used to calculate the VSM signal from a thin film disk.

Three different size thin films were considered, 4 mm, 10mm and 18 mm diameter. The corresponding thickness were 4 micrometers, 10 micrometers and 18 micrometers respectively. The coil response were calculated at the geometric center of the Mallinson coil configuration [7] and were normalized to the response from a point dipole of the same moment as the disks and same gap setting. The response was calculated for various gap spacings (distance between the detection coils) and is shown in fig. 1

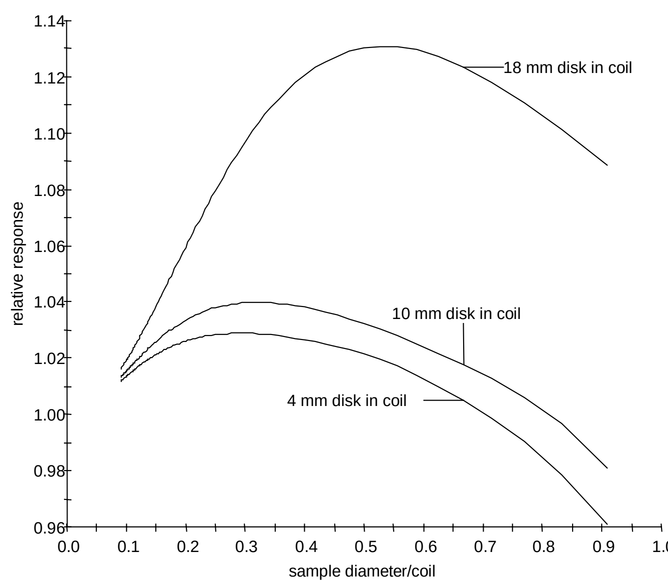

Fig. 1. The VSM coil relative induced voltage of three different size thin film disk for different gap (distance between detection coils) sizes. The data is normalized to a point dipole response of the same moment and gap spacing.

If the coil response were independent of the size of the sample, the curves in Fig. 1 would be straight lines with value 1 (same voltage from the sample as from a point dipole). This implies that the calibration of a VSM can be off by as much as 15% depending on the size of the sample and the gap setting.

The decrease shown in figure 1 as the gap size approaches the diameter of the sample can be understood by the following argument. The highest density of magnetic surface charge will be at the poles of the sample (points nearest the applied magnetic field). As Mallinson [7] pointed out there is an optimal angle between the coils and sample. As the gap is closed on the sample, there is a gap that optimizes the signal level.

III. Conclusions

Large calibration errors can be introduced into a VSM measurement if the VSM is not calibrated with a standard that is the same size and shape of the sample to be measured. The current common procedure of calibrating a VSM with Ni spheres needs to be improved by developing new thin film standards.

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