Standard Curve RX-102A

Standard Curve RX-102A: Measurement Voltage = 1-3 mV DC at T > 1.2 K and at T < 1.2 K we use an AC resistance bridge with excitation of 100 µV ±50% from 0.1 K to 1.2 K and 30 µV ±50% from 0.05 K to 0.1 K

T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd	T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd
0.050	63,765.1	-2,888,654	-2.265	1.40	2,005.18	-666.79	-0.466
0.055	52,106.7	-1,949,063	-2.057	1.60	1,889.42	-504.53	-0.427
0.060	43,515.8	-1,491,859	-2.057	1.80	1,799.38	-402.51	-0.403
0.065	37,123.9	-1,072,213	-1.877	2.00	1,726.40	-330.79	-0.383
0.070	32,601.3	-764,638	-1.642	2.20	1,665.90	-276.51	-0.365
0.075	29,253.3	-592,728	-1.520	2.40	1,615.01	-234.06	-0.348
0.080	26,563.6	-488,024	-1.470	2.60	1,571.69	-200.45	-0.332
0.085	24,330.7	-408,841	-1.428	2.80	1,534.38	-173.60	-0.317
0.090	22,443.7	-348,649	-1.398	3.00	1,501.90	-151.95	-0.304
0.095	20,820.3	-302,798	-1.382	3.20	1,473.34	-134.28	-0.292
0.100	19,400.5	-266,199	-1.372	3.40	1,447.99	-119.66	-0.281
0.110	17,039.1	-209,257	-1.351	3.60	1,425.32	-107.41	-0.271
0.120	15,165.5	-167,396	-1.325	3.80	1,404.90	-97.016	-0.262
0.130	13,658.2	-135,469	-1.289	4.00	1,386.41	-88.095	-0.254
0.140	12,431.6	-110,908	-1.249	4.20	1,369.58	-80.376	-0.246
0.150	11,421.6	-91,897	-1.207	4.40	1,354.20	-73.620	-0.239
0.160	10,579.6	-77,123	-1.166	4.60	1,340.08	-67.671	-0.232
0.170	9,868.60	-65,523	-1.129	4.80	1,327.08	-62.424	-0.226
0.180	9,261.11	-56,333	-1.095	5.00	1,315.07	-57.782	-0.220
0.190	8,735.86	-48,971	-1.065	5.50	1,288.79	-48.114	-0.205
0.200	8,277.06	-43,000	-1.039	6.00	1,266.58	-40.995	-0.194
0.220	7,512.68	-33,971	-0.995	7.00	1,231.55	-29.841	-0.170
0.240	6,900.16	-27,630	-0.961	8.00	1,204.75	-23.747	-0.158
0.260	6,396.63	-22,955	-0.933	9.00	1,183.73	-18.643	-0.142
0.280	5,974.48	-19,408	-0.910	10.0	1,166.92	-15.201	-0.130
0.300	5,615.03	-16,647	-0.889	11.0	1,153.01	-12.727	-0.121
0.320	5,304.98	-14,440	-0.871	12.0	1,141.28	-10.794	-0.113
0.340	5,034.68	-12,647	-0.854	13.0	1,131.30	-9.2310	-0.106
0.360	4,797.02	-11,168	-0.838	14.0	1,122.72	-7.9643	-0.099
0.380	4,586.35	-9,932.3	-0.823	15.0	1,115.29	-6.9393	-0.093
0.400	4,398.49	-8,881.6	-0.808	16.0	1,108.78	-6.1061	-0.088
0.420	4,230.02	-7,990.3	-0.793	17.0	1,103.02	-5.4215	-0.084
0.440	4,078.03	-7,225.7	-0.780	18.0	1,097.90	-4.8514	-0.080
0.460	3,940.33	-6,560.6	-0.766	19.0	1,093.29	-4.3698	-0.076
0.480	3,815.03	-5,983.2	-0.753	20.0	1,089.13	-3.9578	-0.073
0.500	3,700.52	-5,477.7	-0.740	21.0	1,085.36	-3.6010	-0.070
0.550	3,453.50	-4,456.2	-0.710	22.0	1,081.92	-3.2899	-0.067
0.600	3,250.53	-3,698.3	-0.683	23.0	1,078.77	-3.0157	-0.064
0.650	3,080.73	-3,117.8	-0.658	24.0	1,075.87	-2.7734	-0.062
0.700	2,936.57	-2,668.3	-0.636	25.0	1,073.21	-2.5585	-0.060
0.750	2,812.42	-2,307.4	-0.615	26.0	1,070.75	-2.3668	-0.057
0.800	2,704.60	-2,020.0	-0.597	27.0	1,068.47	-2.1955	-0.055
0.850	2,609.10	-1,812.4	-0.590	28.0	1,066.35	-2.0421	-0.054
0.900	2,523.01	-1,626.2	-0.580	29.0	1,064.38	-1.9044	-0.052
0.950	2,446.94	-1,412.1	-0.548	30.0	1,062.54	-1.7805	-0.050
1.000	2,380.77	-1,257.2	-0.528	32.0	1,059.20	-1.5676	-0.047
1.050	2,320.08	-1,170.7	-0.530	34.0	1,056.24	-1.3929	-0.045
1.100	2,263.65	-1,087.4	-0.528	36.0	1,053.61	-1.2483	-0.043
1.150	2,211.30	-1,006.8	-0.524	38.0	1,051.23	-1.1271	-0.041
1.200	2,162.93	-928.26	-0.515	40.0	1,049.08	-1.0253	-0.039

Polynomial Representation

Curve RX-102A can be represented by a polynomial equation based on the Chebycheb polynomials which are described below. Three separate ranges are required to accurately describe the curve, with the parameters for these ranges given in Table 1. The polynomials represent Curve RX-102A on the preceding page with RMS deviations on the order of ±0.5 mK below 1 K, ±2 mK below 6 K, ±7 mK below 20 K, and ±35 mK below 40 K.

The Chebycheb equation is of the form:

$$T(x) = \sum_{i=0}^{n} a_i t_i(x) \qquad (1)$$

Where T(x) represents the temperature in kelvin, $t_i(x)$ is a Chebycheb polynomial, and a_i represents the Chebycheb coefficients. The parameter x is a normalized variable given by:

$$x = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \qquad (2)$$

where Z is the log (base 10) of the resistance and ZL and ZU designate the log of the lower and upper limit of the resistance over the fit range.

The Chebychev polynomials can be generated from the recursion relation:

$$t_{i+1}(x) = 2xt_i(x) - t_{i-1}(x) \qquad (3)$$

$$t_0 = 1, \quad t_1(x) = x$$

Alternatively, these polynomials are given by:

$$t_1(x) = \cos\left[i \times \arccos(x)\right] \qquad (4)$$

The use of Chebycheb polynomials is no more complicated than the use of the regular power series and the offer significant advantages in the actual fitting process. The first step is to transform the measured voltage into the normalized variable using equation 2. Equation 1 is then used in combination with Equations 3 and 4 to calculate the temperature. Programs 1 and 2 provide sample BASIC subroutines which will take the resistance and return the temperature T calculated from Chebychev fits. The subroutines assume the values ZL and ZU have been input along with the degree of the fit. The Chebycheb coefficients are also assumed to be in an array A(0), A(1), …, A(Ndegree).

An interesting property of the Chebychev fit is evident in the form of the Chebycheb polynomial given in Equation 4. No term in Equation 1 will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if the full accuracy is not required.

Table 1. Chebychev fit coefficients.

```
Fit Range:  0.050 K to 0.950 K
Order = 8
A(0)= 0.300923     ZL=3.35453159798
A(1)=-0.401714     ZU=5.00000000000
A(2)= 0.220055
A(3)=-0.098891
A(4)= 0.046804
A(5)=-0.017379
A(6)= 0.009090
A(7)=-0.002703
A(8)= 0.002170


Fit Range:  0.950 K to 6.5 K
Order = 9
A(0)= 2.813252     ZL=3.08086045368
A(1)=-2.976371     ZU=3.44910010859
A(2)= 1.299095
A(3)=-0.538334
A(4)= 0.220456
A(5)=-0.090969
A(6)= 0.037095
A(7)=-0.015446
A(8)= 0.005104
A(9)=-0.004254


Fit Range:  6.5 K to 40 K
Order = 9
A(0)= 3074.395992  ZL=2.95500000000
A(1)=-5680.735415  ZU=3.10855552727
A(2)= 4510.873058
A(3)=-3070.206226
A(4)= 1775.293345
A(5)= -857.606658
A(6)=  336.220971
A(7)= -101.617491
A(8)=   21.390256
A(9)=   -2.407847
```

Program 2. BASIC subroutine for evaluating the the temperature T from the Chebychev series using Equations 1 and 3. An array Tc(Ndegree) must be defined.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   Tc(0)=1
   Tc(1)=X
   T=A(0)+A(1)*X
   FOR I=2 to Ndegree
      Tc(I)=2*X*Tc(I-1)-Tc(I-2)
      T=T+A(I)*Tc(I)
   NEXT I
   RETURN
```

Program 2. BASIC subroutine for evaluating the temperature T from the Chebychev series using Equations 1 and 4.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   T=0
   FOR I=0 to Ndegree
      T=T+A(I)*COS(I*ARCCOS(X))
   NEXT I
   RETURN
```