Standard Curve RX-103A

Standard Curve RX-103A: Measurement Voltage = 1-3 mV DC at T > 1.2 K and at T < 1.2 K we use an AC resistance bridge with excitation of 100 µV ±50% from 0.1 K to 1.2 K and 30 µV ±50% from 0.05 K to 0.1 K

T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd	T (K)	Resistance (ohms)	dr/dt (ohms/K)	Sd
1.20	33,968.2	-19,310	-0.682	9.00	14,452.7	-405.1	-0.252
1.30	32,214.0	-15,954	-0.644	9.50	14,259.1	-369.7	-0.246
1.40	30,745.1	-13,571	-0.618	10.0	14,082.6	-336.8	-0.239
1.50	29,479.8	-11,792	-0.600	10.5	13,921.8	-307.0	-0.232
1.60	28,376.3	-10,327	-0.582	11.0	13,775.0	-281.0	-0.224
1.70	27,406.0	-9,116	-0.565	11.5	13,640.3	-258.3	-0.218
1.80	26,546.3	-8,110	-0.550	12.0	13,516.2	-238.6	-0.212
1.90	25,778.6	-7,267	-0.536	12.5	13,401.3	-221.4	-0.207
2.00	25,088.7	-6,551	-0.522	13.0	13,294.5	-206.4	-0.202
2.10	24,464.9	-5,939	-0.510	13.5	13,194.6	-193.2	-0.198
2.20	23,898.0	-5,412	-0.498	14.0	13,101.0	-181.5	-0.194
2.30	23,380.3	-4,953	-0.487	14.5	13,012.9	-171.1	-0.191
2.40	22,905.4	-4,552	-0.477	15.0	12,929.7	-161.8	-0.188
2.50	22,468.2	-4,200	-0.467	15.5	12,850.9	-153.3	-0.185
2.60	22,064.1	-3,888	-0.458	16.0	12,776.2	-145.6	-0.182
2.70	21,689.3	-3,611	-0.450	16.5	12,705.2	-138.6	-0.180
2.80	21,340.8	-3,364	-0.441	17.0	12,637.5	-132.2	-0.178
2.90	21,015.7	-3,143	-0.434	17.5	12,572.9	-126.2	-0.176
3.00	20,711.5	-2,943	-0.426	18.0	12,511.2	-120.7	-0.174
3.10	20,426.3	-2,764	-0.419	18.5	12,452.1	-115.6	-0.172
3.20	20,158.3	-2,600	-0.413	19.0	12,395.5	-110.8	-0.170
3.30	19,905.7	-2,452	-0.407	19.5	12,341.2	-106.4	-0.168
3.40	19,667.4	-2,317	-0.401	20.0	12,289.1	-102.2	-0.166
3.50	19,442.0	-2,193	-0.395	20.5	12239.0	-98.28	-0.165
3.60	19,228.4	-2,080	-0.389	21.0	12,190.8	-94.57	-0.163
3.70	19,025.7	-1,975	-0.384	22.0	12,099.7	-87.79	-0.160
3.80	18,833.1	-1,879	-0.379	23.0	12,015.0	-81.74	-0.156
3.90	18,649.7	-1,790	-0.374	24.0	11,936.0	-76.30	-0.153
4.00	18,474.9	-1,707	-0.370	25.0	11,862.2	-71.40	-0.150
4.20	18,148.7	-1,559	-0.361	26.0	11,793.0	-66.97	-0.148
4.40	17,850.0	-1,430	-0.353	27.0	11,728.1	-62.96	-0.145
4.60	17,575.4	-1,318	-0.345	28.0	11,667.0	-59.31	-0.142
4.80	17,322.1	-1,218	-0.338	29.0	11,609.4	-55.99	-0.140
5.00	17,087.4	-1,130	-0.331	30.0	11,554.9	-52.96	-0.137
5.20	16,869.3	-1,052	-0.324	31.0	11,503.4	-50.18	-0.135
5.40	16,666.1	-981.8	-0.318	32.0	11,454.5	-47.63	-0.133
5.60	16,476.1	-918.5	-0.312	33.0	11,408.0	-45.30	-0.131
5.80	16,298.2	-861.6	-0.307	34.0	11,363.8	-43.15	-0.129
6.00	16,131.0	-812.0	-0.302	35.0	11,321.7	-41.17	-0.127
6.50	15,752.8	-703.0	-0.290	36.0	11,281.5	-39.34	-0.126
7.00	15,426.2	-605.4	-0.275	37.0	11,243.0	-37.63	-0.124
7.50	15,144.1	-527.5	-0.261	38.0	11,206.1	-36.06	-0.122
8.00	14,894.0	-477.3	-0.256	39.0	11,170.8	-34.60	-0.121
8.50	14,664.3	-441.4	-0.256	40.0	11,136.9	-33.24	-0.119

Polynomial Representation

Curve RX-103 can be represented by a polynomial equation based on the Chebycheb polynomials which are described below. Three separate ranges are required to accurately describe the curve, with the parameters for these ranges given in Table 1. The polynomials represent Curve RX-103 on the preceding page with RMS deviations on the order of ±1 mK below 8 K, ±30 mK below 20 K, ±100 mK below 30 K, and ±1 K below 40 K.

The Chebycheb equation is of the form:

$$T(x) = \sum_{i=0}^{n} a_i t_i(x) \qquad (1)$$

Where T(x) represents the temperature in kelvin, $t_i(x)$ is a Chebycheb polynomial, and a_i represents the Chebycheb coefficients. The parameter x is a normalized variable given by:

$$x = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \qquad (2)$$

where Z is the LOG (base 10) of the resistance and ZL and ZU designate the log of the lower and upper limit of the resistance over the fit range.

The Chebychev polynomials can be generated from the recursion relation:

$$t_{i+1}(x) = 2xt_i(x) - t_{i-1}(x) \qquad (3)$$

$$t_0 = 1, \quad t_1(x) = x$$

Alternatively, these polynomials are given by:

$$t_1(x) = \cos\left[i \times \arccos(x)\right] \qquad (4)$$

The use of Chebycheb polynomials is no more complicated than the use of the regular power series and the offer significant advantages in the actual fitting process. The first step is to transform the measured voltage into the normalized variable using equation 2. Equation 1 is then used in combination with Equations 3 and 4 to calculate the temperature. Programs 1 and 2 provide sample BASIC subroutines which will take the resistance and return the temperature T calculated from Chebychev fits. The subroutines assume the values ZL and ZU have been input along with the degree of the fit. The Chebycheb coefficients are also assumed to be in an array A(0), A(1), …, A(Ndegree).

An interesting property of the Chebychev fit is evident in the form of the Chebycheb polynomial given in Equation 4. No term in Equation 1 will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if the full accuracy is not required.

Table 1. Chebychev fit coefficients.

```
Fit Range:   1.20 K to 8.00 K
Order = 8
A(0)= 3.867813     ZL=4.14870225626
A(1)=-2.699925     ZU=4.70000000000
A(2)= 2.660639
A(3)=-0.039368
A(4)= 0.730964
A(5)= 0.142597
A(6)= 0.157014
A(7)=-0.029147
A(8)= 0.016263


Fit Range:  8.00 K to 40.0 K
Order = 6
A(0)=  95.856570   ZL=3.95400000000
A(1)=-148.658361   ZU=4.20766712797
A(2)=  89.788569
A(3)= -44.216666
A(4)=  17.489295
A(5)=  -5.183444
A(6)=   0.928979
```

Program 2. BASIC subroutine for evaluating the the temperature T from the Chebychev series using Equations 1 and 3. An array Tc(Ndegree) must be defined.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   Tc(0)=1
   Tc(1)=X
   T=A(0)+A(1)*X
   FOR I=2 to Ndegree
      Tc(I)=2*X*Tc(I-1)-Tc(I-2)
      T=T+A(I)*Tc(I)
   NEXT I
   RETURN
```

Program 2. BASIC subroutine for evaluating the temperature T from the Chebychev series using Equations 1 and 4.

```
REM Evaluation of Chebychev series
   x=((Z-ZL)-ZU-Z))/(ZU-ZL)
   T=0
   FOR I=0 to Ndegree
      T=T+A(I)*COS(I*ARCCOS(X))
   NEXT I
   RETURN
```