

## Standard Curve DT-670: Measurement Current = 10 $\mu$ A $\pm$ 0.05%

T (K)	Voltage (V)	dV/dT (mV/K)	T (K)	Voltage (V)	dV/dT (mV/K)	T (K)	Voltage (V)	dV/dT (mV/K)
1.20	1.646540	-9.87	18.00	1.228300	-15.25	125.00	0.939242	-1.96
1.40	1.644290	-12.49	18.50	1.220700	-15.18	130.00	0.929390	-1.98
1.60	1.641570	-14.79	19.00	1.213110	-15.20	135.00	0.919446	-2.00
1.80	1.638370	-17.15	19.50	1.205480	-15.34	140.00	0.909416	-2.01
2.00	1.634720	-19.30	20.00	1.197748	-15.63	145.00	0.899304	-2.03
2.20	1.630670	-21.14	21.00	1.181548	-16.98	150.00	0.889114	-2.05
2.40	1.626290	-22.61	22.00	1.162797	-21.11	155.00	0.878851	-2.06
2.60	1.621660	-23.63	23.00	1.140817	-20.77	160.00	0.868518	-2.07
2.80	1.616870	-24.16	24.00	1.125923	-9.42	165.00	0.858120	-2.09
3.00	1.612000	-24.67	25.00	1.119448	-4.60	170.00	0.847659	-2.10
3.20	1.606970	-25.63	26.00	1.115658	-3.19	175.00	0.837138	-2.11
3.40	1.601730	-26.80	27.00	1.112810	-2.58	180.00	0.826560	-2.12
3.60	1.596260	-27.91	28.00	1.110421	-2.25	185.00	0.815928	-2.13
3.80	1.590570	-28.99	29.00	1.108261	-2.08	190.00	0.805242	-2.14
4.00	1.584650	-30.21	30.00	1.106244	-1.96	195.00	0.794505	-2.15
4.20	1.578480	-31.59	31.00	1.104324	-1.88	200.00	0.783720	-2.16
4.40	1.572020	-32.91	32.00	1.102476	-1.82	210.00	0.762007	-2.18
4.60	1.565330	-33.97	33.00	1.100681	-1.77	220.00	0.740115	-2.20
4.80	1.558450	-34.74	34.00	1.098930	-1.73	230.00	0.718054	-2.21
5.00	1.551450	-35.25	35.00	1.097216	-1.70	240.00	0.695834	-2.23
5.20	1.544360	-35.60	36.00	1.095534	-1.67	250.00	0.673462	-2.24
5.40	1.537210	-35.92	37.00	1.093878	-1.64	260.00	0.650949	-2.26
5.60	1.530000	-36.22	38.00	1.092244	-1.62	270.00	0.628302	-2.27
5.80	1.522730	-36.48	39.00	1.090627	-1.61	273.15	0.621141	-2.28
6.00	1.515410	-36.71	40.00	1.089024	-1.60	280.00	0.605528	-2.28
6.50	1.496980	-36.86	42.00	1.085842	-1.59	290.00	0.582637	-2.29
7.00	1.478680	-36.21	44.00	1.082669	-1.59	300.00	0.559639	-2.30
7.50	1.460860	-35.00	46.00	1.079492	-1.59	305.00	0.548102	-2.31
8.00	1.443740	-33.42	48.00	1.076303	-1.60	310.00	0.536542	-2.31
8.50	1.427470	-31.67	50.00	1.073099	-1.61	320.00	0.513361	-2.32
9.00	1.412070	-29.95	52.00	1.069881	-1.61	330.00	0.490106	-2.33
9.50	1.397510	-28.31	54.00	1.066650	-1.62	340.00	0.466760	-2.34
10.00	1.383730	-26.84	56.00	1.063403	-1.63	350.00	0.443371	-2.34
10.50	1.370650	-25.51	58.00	1.060141	-1.64	360.00	0.419960	-2.34
11.00	1.358200	-24.31	60.00	1.056862	-1.64	370.00	0.396503	-2.35
11.50	1.346320	-23.20	65.00	1.048584	-1.67	380.00	0.373002	-2.35
12.00	1.334990	-22.15	70.00	1.040183	-1.69	390.00	0.349453	-2.36
12.50	1.324160	-21.17	75.00	1.031651	-1.72	400.00	0.325839	-2.36
13.00	1.313810	-20.25	77.35	1.027594	-1.73	410.00	0.302161	-2.37
13.50	1.303900	-19.41	80.00	1.022984	-1.75	420.00	0.278416	-2.38
14.00	1.294390	-18.63	85.00	1.014181	-1.77	430.00	0.254592	-2.39
14.50	1.285260	-17.92	90.00	1.005244	-1.80	440.00	0.230697	-2.39
15.00	1.276450	-17.31	95.00	0.996174	-1.83	450.00	0.206758	-2.39
15.50	1.267940	-16.77	100.00	0.986974	-1.85	460.00	0.182832	-2.39
16.00	1.259670	-16.31	105.00	0.977650	-1.88	470.00	0.159010	-2.37
16.50	1.251610	-15.94	110.00	0.968209	-1.90	480.00	0.135480	-2.33
17.00	1.243720	-15.64	115.00	0.958657	-1.92	490.00	0.112553	-2.25
17.50	1.235960	-15.41	120.00	0.949000	-1.94	500.00	0.090681	-2.12

## POLYNOMIAL REPRESENTATION

Curve DT-670 can be expressed by a polynomial equation based on the Chebychev polynomials. Four separate ranges are required to accurately describe the curve. Table 1 lists the parameters for these ranges. The polynomials represent Curve DT-670 on the preceding page with RMS deviations of 10 mK. The Chebychev equation is:

$$T(x) = \sum_{i=0}^n a_i t_i(x) \quad (1)$$

where  $T(x)$  = temperature in Kelvin,  $t_i(x)$  = a Chebychev polynomial, and  $a_i$  = the Chebychev coefficient. The parameter  $x$  is

$$x = \frac{(Z - ZL) - (ZU - Z)}{(ZU - ZL)} \quad (2)$$

where  $Z$  = voltage and  $ZL$  and  $ZU$  = lower and upper limit of the voltage over the fit range. The Chebychev polynomials can

$$t_{i+1}(x) = 2xt_i(x) - t_{i-1}(x) \quad (3)$$

be generated from the recursion relation:

$$t_0(x) = 1, \quad t_1(x) = x$$

Alternately, these polynomials are given by:

$$t_i(x) = \cos[i \cdot \arccos(x)] \quad (4)$$

The use of Chebychev polynomials is no more complicated than the use of the regular power series and they offer significant advantages in the actual fitting process. The first step is to transform the measured voltage into the normalized variable using Equation (2). Equation (1) is then used in combination with equations (3) and (4) to calculate the temperature. Programs 1 and 2 provide sample BASIC subroutines which will take the voltage and return the temperature  $T$  calculated from Chebychev fits. The subroutines assume the values  $ZL$  and  $ZU$  have been input along with the degree of the fit. The Chebychev coefficients are also assumed to be in any array  $A(0), A(1), \dots, A(i_{\text{degree}})$ .

An interesting property of the Chebychev fit is evident in the form of the Chebychev polynomial given in Equation (4). No term in Equation (1) will be greater than the absolute value of the coefficient. This property makes it easy to determine the contribution of each term to the temperature calculation and where to truncate the series if full accuracy is not required.

```

FUNCTION Chebychev (Z as double) as
double
REM Evaluation of Chebychev series
X = ((Z - ZL) - (ZU - Z)) / (ZU - ZL)
Tc(0) = 1
Tc(1) = X
T = A(0) + A(1) * X
FOR I = 2 to Ubound(A())
    Tc(I) = 2 * X * Tc(I-1) - Tc(I-2)
    T = T + A(I) * Tc(I)
NEXT I
Chebychev = T
END FUNCTION
    
```

**Program 1.** BASIC subroutine for evaluating the temperature  $T$  from the Chebychev series using Equations (1) and (3). An array  $T_c(i_{\text{degree}})$  should be dimensioned. See text for details.

```

FUNCTION Chebychev (Z as double) as
double
REM Evaluation of Chebychev series
X = ((Z - ZL) - (ZU - Z)) / (ZU - ZL)
T = 0
FOR I = 0 to Ubound(A())
    T = T + A(I) * COS(I * ARCCOS(X))
NEXT I
Chebychev = T
END FUNCTION
    
```

**NOTE:**  $\arccos(X) = \frac{\pi}{2} - \arctan\left[\frac{X}{\sqrt{1-X^2}}\right]$

**Program 2.** BASIC subroutine for evaluating the temperature  $T$  from the Chebychev series using Equations (1) and (4). Double precision calculations are recommended.

**Table 1**

2.0 K to 12.0 K	12.0 K to 24.5 K	24.5 K to 100.0 K	100 K to 500 K
ZL = 1.294390	ZL = 1.11230	ZL = 0.909416	ZL = 0.07000
ZU = 1.680000	ZU = 1.38373	ZU = 1.122751	ZU = 0.99799
A(0) = 6.429274	A(0) = 17.244846	A(0) = 82.017868	A(0) = 306.592351
A(1) = -7.514262	A(1) = -7.964373	A(1) = -59.064244	A(1) = -205.393808
A(2) = -0.725882	A(2) = 0.625343	A(2) = -1.356615	A(2) = -4.695680
A(3) = -1.117846	A(3) = -0.105068	A(3) = 1.055396	A(3) = -2.031603
A(4) = -0.562041	A(4) = 0.292196	A(4) = 0.837341	A(4) = -0.071792
A(5) = -0.360239	A(5) = -0.344492	A(5) = 0.431875	A(5) = -0.437682
A(6) = -0.229751	A(6) = 0.271670	A(6) = 0.440840	A(6) = 0.176352
A(7) = -0.135713	A(7) = -0.151722	A(7) = -0.061588	A(7) = -0.182516
A(8) = -0.068203	A(8) = 0.121320	A(8) = 0.209414	A(8) = 0.064687
A(9) = -0.029755	A(9) = -0.035566	A(9) = -0.120882	A(9) = -0.027019
	A(10) = 0.045966	A(10) = 0.055734	A(10) = 0.010019
		A(11) = -0.035974	